

On the identification of vertices in graphs using paths ^{*}

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A set $\mathcal{C} = \{C_1, C_2, \dots, C_k\}$ of subgraphs of a graph G is said to identify the vertices of G if all sets $I(v, \mathcal{C}) = \{i \mid v \in C_i\}$ are nonempty and for any pair of distinct vertices u, v of G , $I(u, \mathcal{C}) \neq I(v, \mathcal{C})$. The idea here is that from knowing all sets $I(v, \mathcal{C})$ we can uniquely determine the vertices of G .

If we restrict \mathcal{C} to be a subset of a given family \mathcal{G} of subgraphs of G , we denote by $ID_{\mathcal{G}}(G)$, the minimum number of subgraphs of G from \mathcal{G} needed to identify all vertices of G . If we take as \mathcal{G} to be the family of all maximal stars of G , we get the definition of identifying codes. The case when \mathcal{G} equals the family of all cycles or closed walks of G , has been studied for special graph classes in [1, 2, 3].

In this talk, we present some results when \mathcal{G} equals the family \mathcal{P} of all paths of a graph G . In particular, we determine $ID_{\mathcal{P}}(T)$ for any tree T in terms of the number of leaves and the number of vertices of degree 2. For a path on n vertices, P_n , $ID_{\mathcal{P}}(P_n) = \lceil \frac{n+1}{2} \rceil$. For trees without vertices of degree 2 we have $ID_{\mathcal{P}}(T) = \lceil \frac{2l}{3} \rceil$. When a tree T has also vertices of degree 2 more cases appear and we present all of them.

Results about the extremal cases in terms of the number of vertices for the value of $ID_{\mathcal{P}}(G)$ for general graphs will also be discussed.

References

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