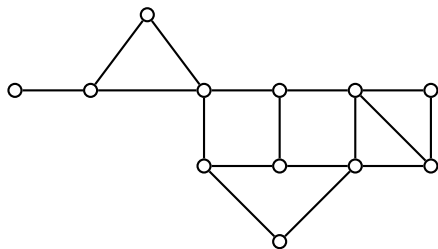


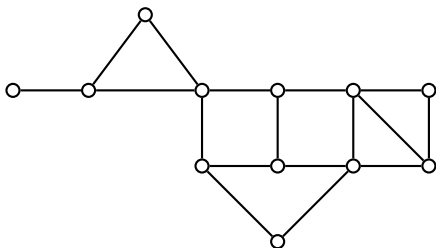
# Sufficient conditions for the existence of two disjoint independent dominating sets

Oliver Schaudt

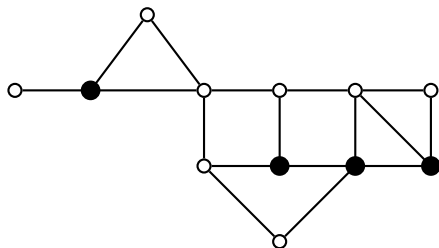
Department for Computer Science  
Group Faigle/Schrader  
University of Cologne

BWIC 2011

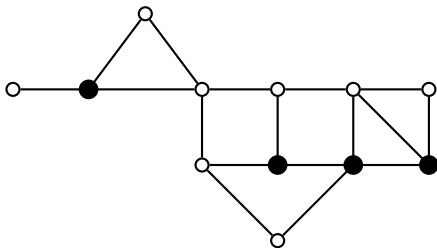




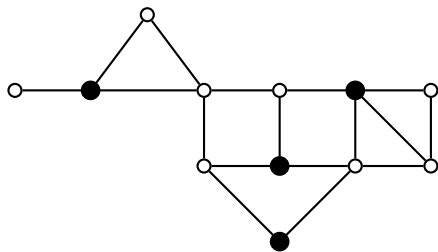
- A **dominating set** is a vertex subset such that any vertex outside of the subset has a neighbor in the subset



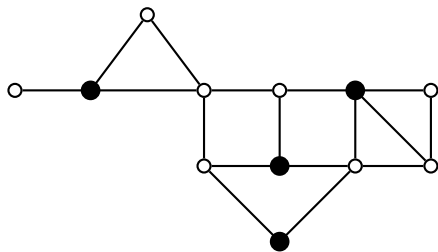
- A **dominating set** is a vertex subset such that any vertex outside of the subset has a neighbor in the subset



- A **dominating set** is a vertex subset such that any vertex outside of the subset has a neighbor in the subset
- An **independent dominating set** is dominating set that is also an independent set



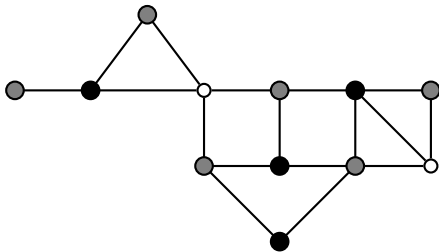
- A **dominating set** is a vertex subset such that any vertex outside of the subset has a neighbor in the subset
- An **independent dominating set** is dominating set that is also an independent set



- A **dominating set** is a vertex subset such that any vertex outside of the subset has a neighbor in the subset
- An **independent dominating set** is dominating set that is also an independent set

## Problem

In a given graph, are there two disjoint independent dominating sets?



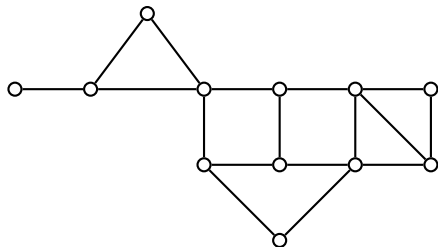
- A **dominating set** is a vertex subset such that any vertex outside of the subset has a neighbor in the subset
- An **independent dominating set** is dominating set that is also an independent set

## Problem

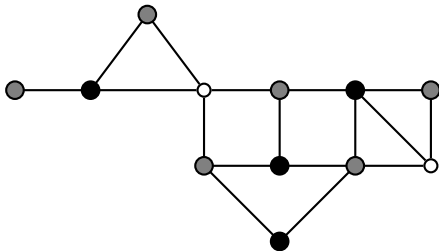
In a given graph, are there two disjoint independent dominating sets?



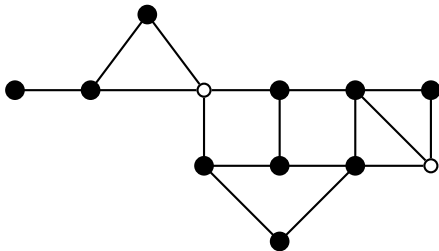
# An equivalent condition



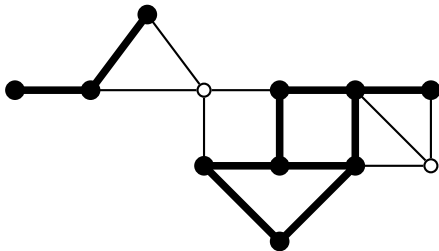
# An equivalent condition



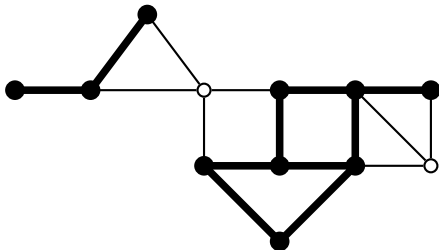
# An equivalent condition



# An equivalent condition

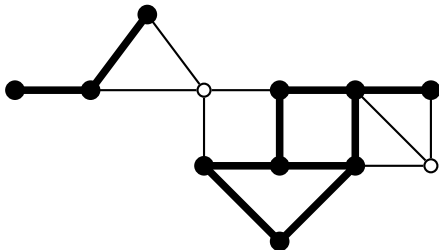


# An equivalent condition



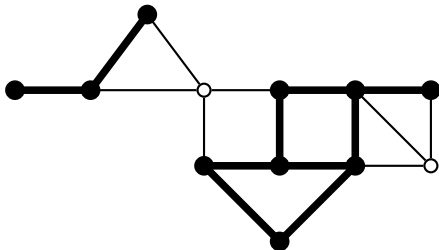
- the union of two disjoint independent dominating sets ...

# An equivalent condition



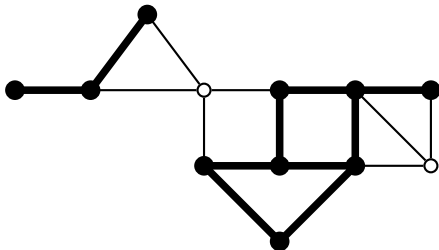
- the union of two disjoint independent dominating sets ...
- is a dominating set without isolated vertices (a **total dominating set**)

# An equivalent condition



- the union of two disjoint independent dominating sets ...
- is a dominating set without isolated vertices (a **total dominating set**)
- its induced subgraph is bipartite

# An equivalent condition



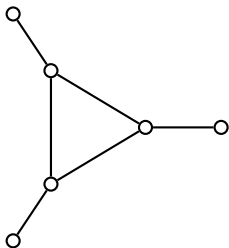
- the union of two disjoint independent dominating sets ...
- is a dominating set without isolated vertices (a **total dominating set**)
- its induced subgraph is bipartite

## Lemma

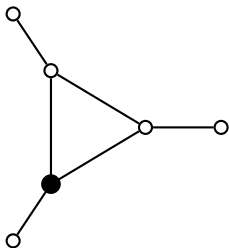
A graph has two disjoint independent dominating sets *iff* it has a bipartite total dominating set.



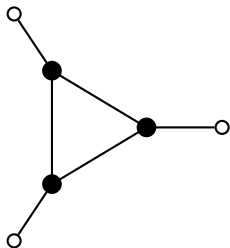
# Minimal graphs without bipartite total dominating sets



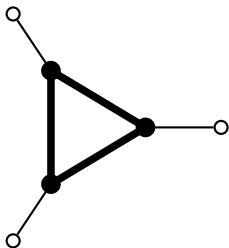
# Minimal graphs without bipartite total dominating sets



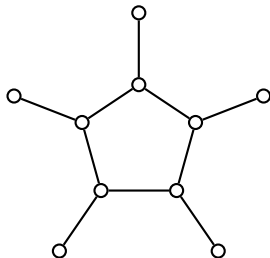
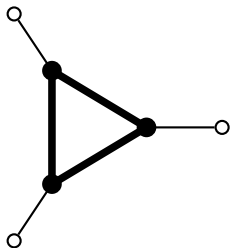
# Minimal graphs without bipartite total dominating sets



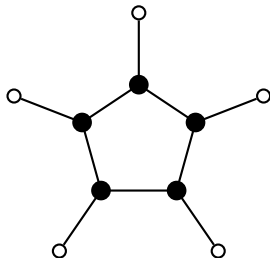
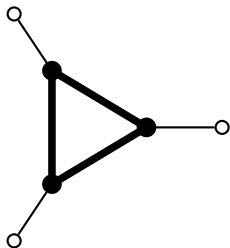
# Minimal graphs without bipartite total dominating sets



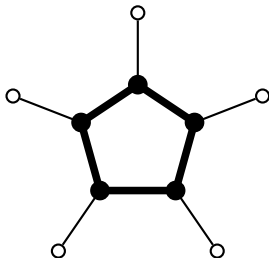
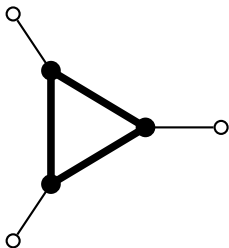
# Minimal graphs without bipartite total dominating sets

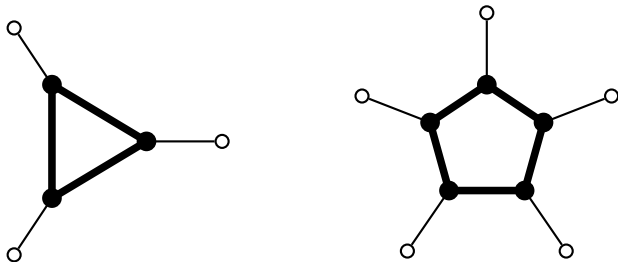


# Minimal graphs without bipartite total dominating sets



# Minimal graphs without bipartite total dominating sets

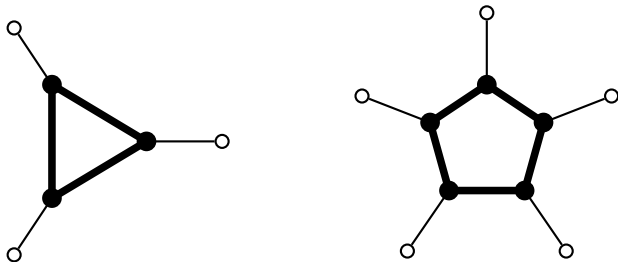




- coronas of chordless odd cycles (**odd coronas**) are minimal graphs without bipartite total dominating sets

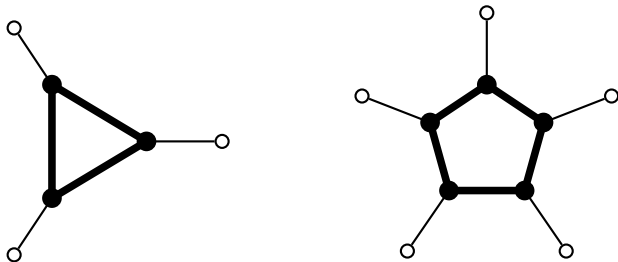


# Minimal graphs without bipartite total dominating sets



- coronas of chordless odd cycles (**odd coronas**) are minimal graphs without bipartite total dominating sets
- Structural domination theorems [Bacsó, Tuza, S.]  $\rightsquigarrow$  there are no other minimal graphs without bipartite total dominating sets

# Minimal graphs without bipartite total dominating sets



- coronas of chordless odd cycles (**odd coronas**) are minimal graphs without bipartite total dominating sets
- Structural domination theorems [Bacsó, Tuza, S.]  $\rightsquigarrow$  there are no other minimal graphs without bipartite total dominating sets

## Theorem

Every odd-corona-free graph has two disjoint independent dominating sets.

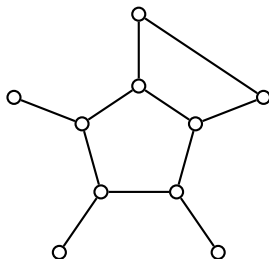
## Theorem

There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.

# Finding two disjoint independent dominating sets

## Theorem

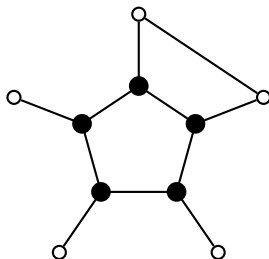
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

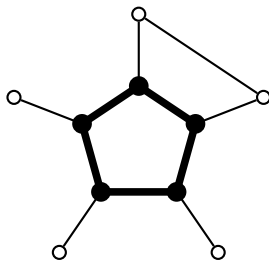
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

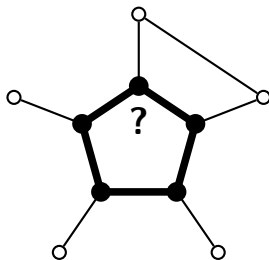
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

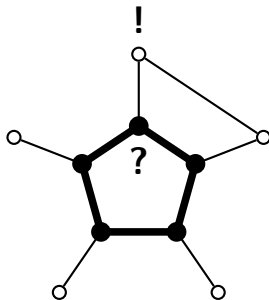
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.

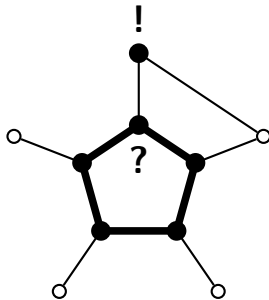




# Finding two disjoint independent dominating sets

## Theorem

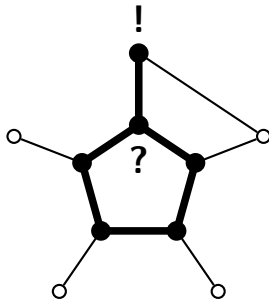
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

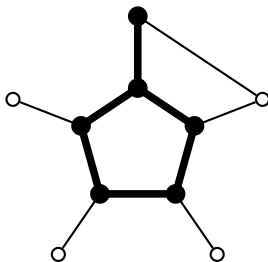
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

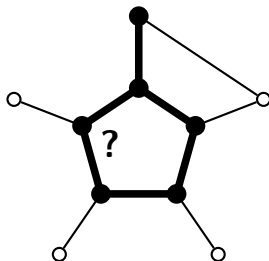
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

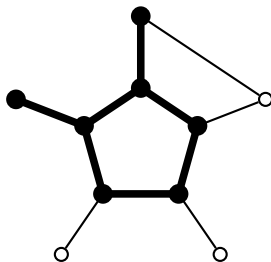
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

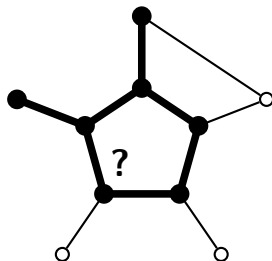
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

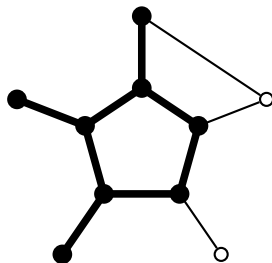
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

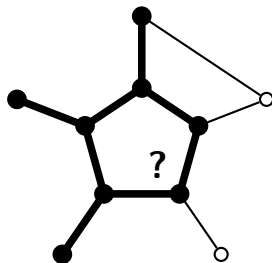
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.

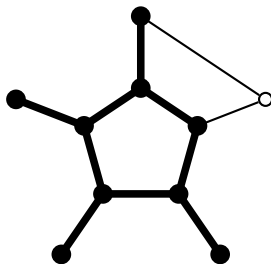




# Finding two disjoint independent dominating sets

## Theorem

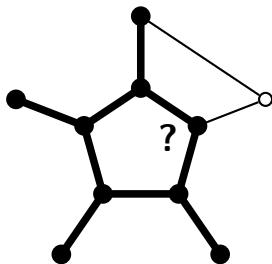
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

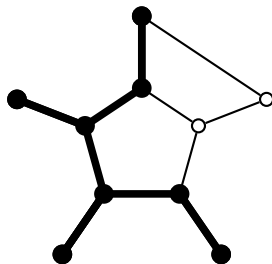
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

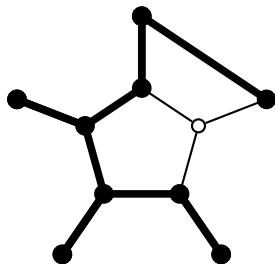
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

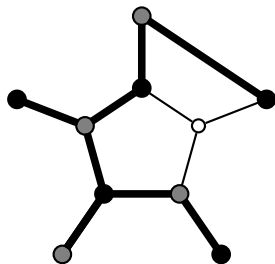
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

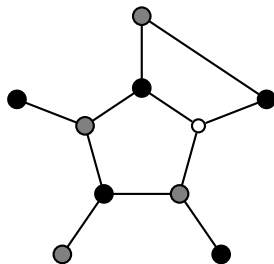
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

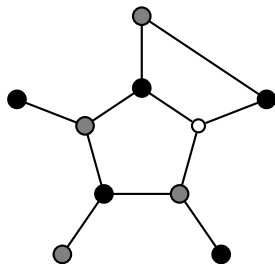
There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



# Finding two disjoint independent dominating sets

## Theorem

There is a polynomial algorithm that finds, for any input graph, either two disjoint independent dominating sets or an induced odd corona.



## Corollary

Every odd-corona-free graph has two disjoint independent dominating sets and such sets can be computed in polynomial time.

# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges



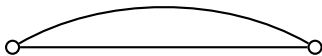
# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges



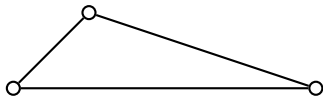
# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges



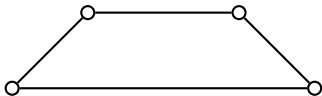
# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges



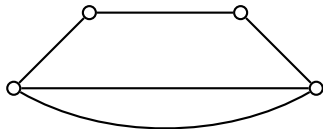
# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges



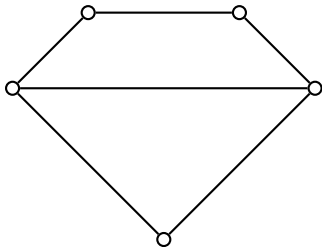
# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges



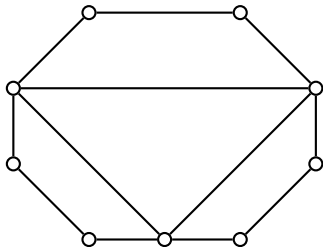
# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges



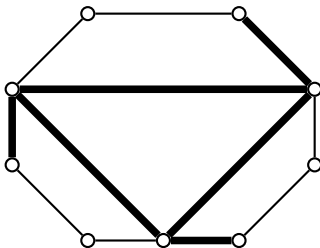
# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges



# Series-parallel graphs

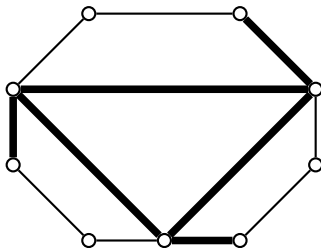
- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges





# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges

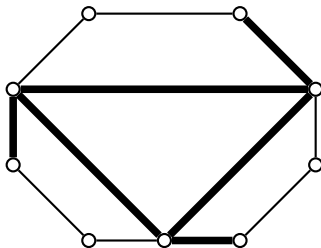


## Theorem

Every series-parallel graph has two disjoint independent dominating sets.

# Series-parallel graphs

- a multigraph is called **series-parallel** if it is obtained from  $K_2$  by edge subdivisions and / or addition of parallel edges



## Theorem

Every series-parallel graph has two disjoint independent dominating sets.

**Thanks!**