

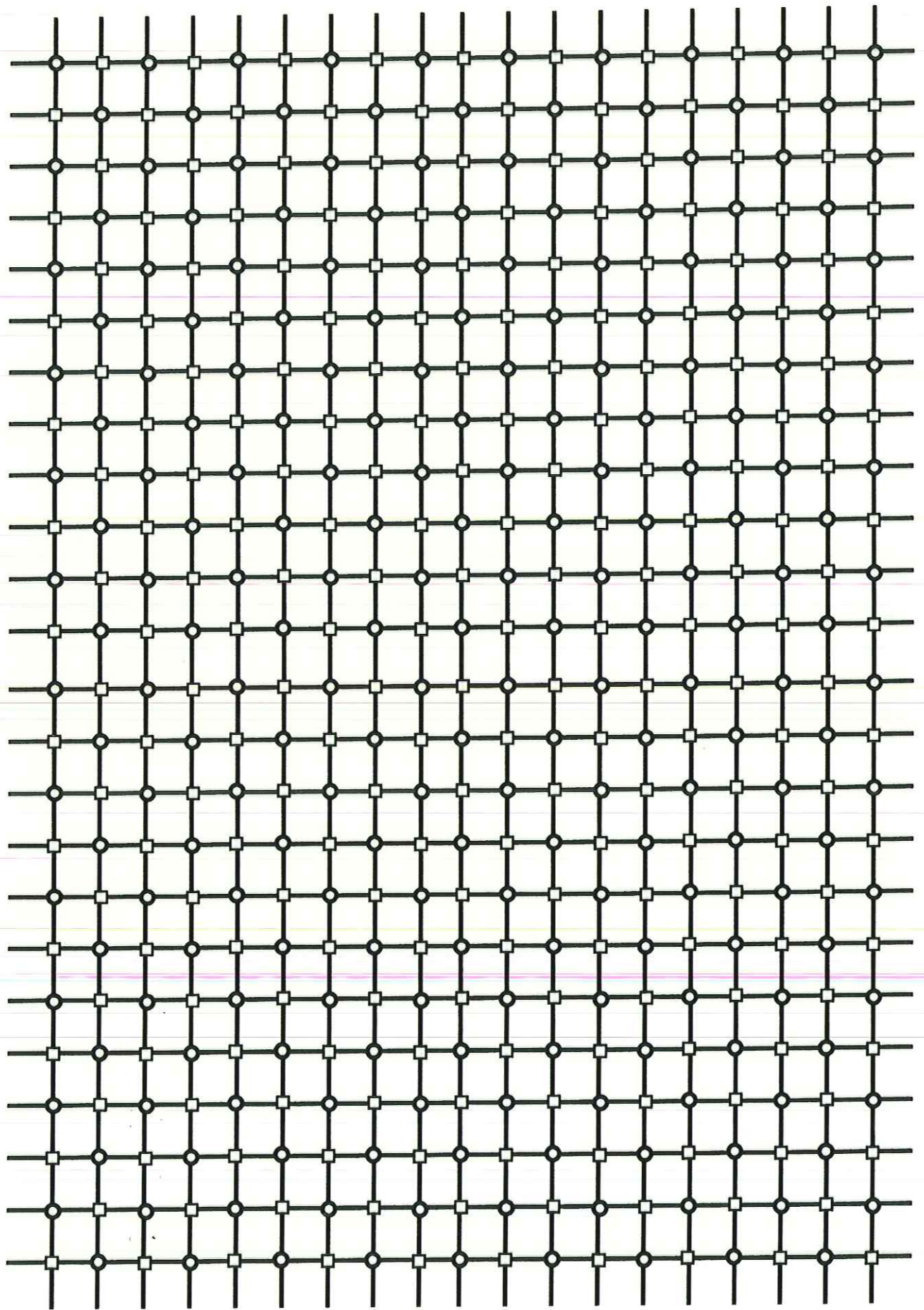
FACTS ARE
STUBBORN THINGS

Rene le Sage

THE COWARDS NEVER
STARTED, AND THE WEAK
DIED ON THE WAY.

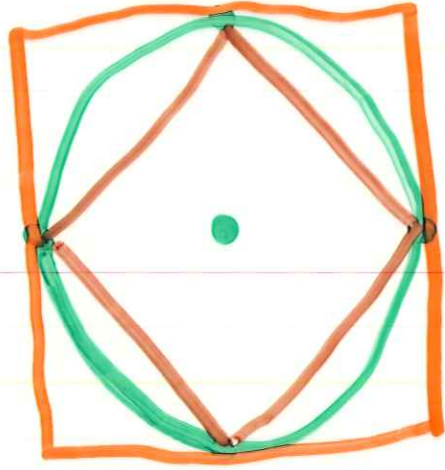
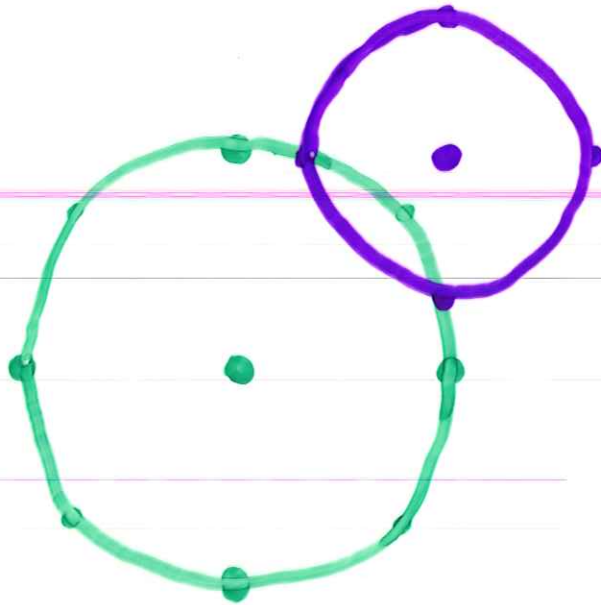
Idit Carson





(x_1, y_1)

(x_2, y_2)



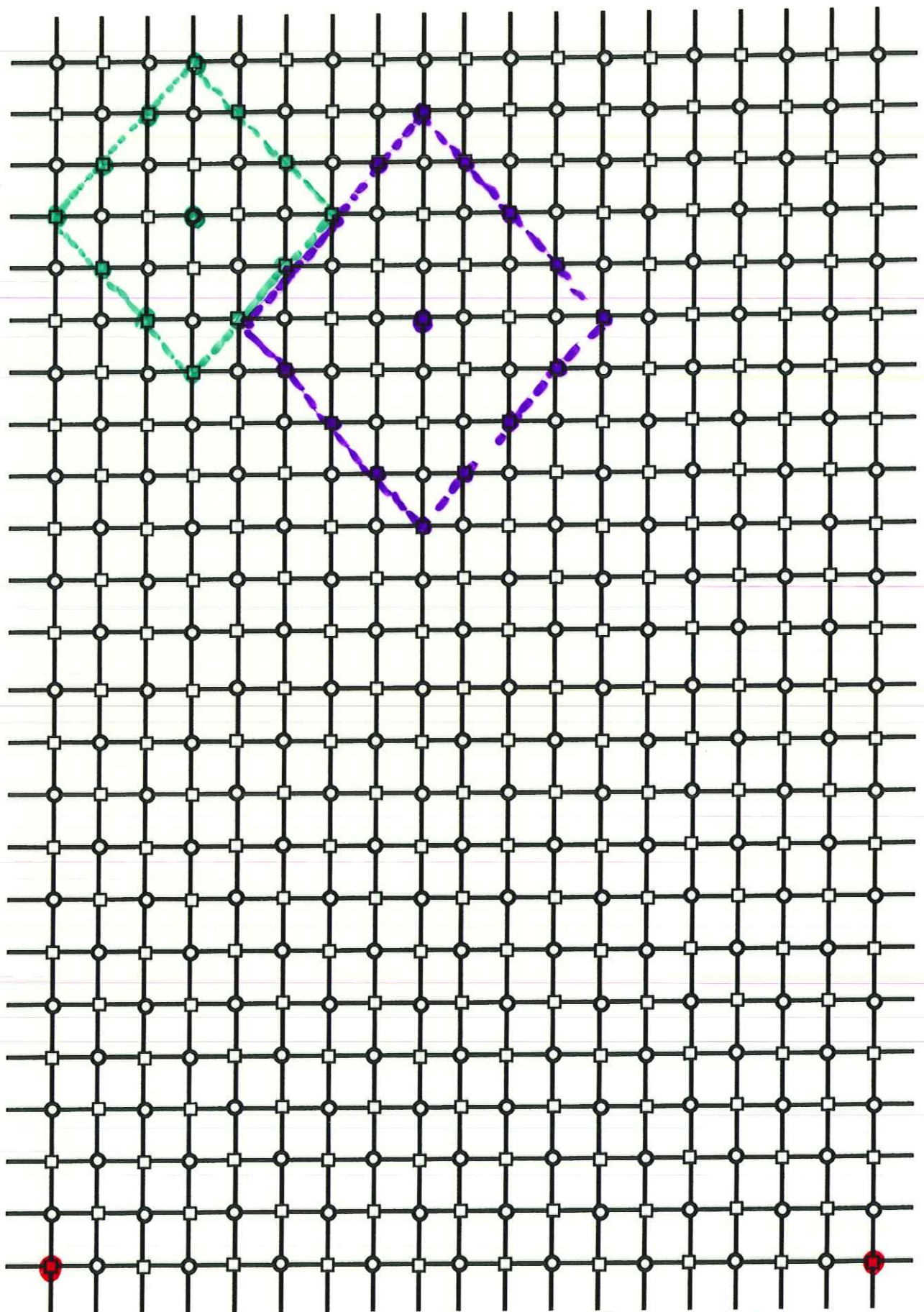
l_1

l_2

l_0

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\log_2(\mathbb{R}^2) = 3$$



LOC, (Z □ Z) = ?

branch paths is homeomorphic to $K_{1,n}$. Note that distinct leaves of T must be disjoint. Figure 1 exhibits four topologically distinct trees which have five leaves with stem points labelled v_1, v_2, v_3, v_4 and v_5 . In each of them, v_2 has $\deg(v_2) - 1 = 3$ branch paths. Every tree which is not a path has at least one leaf, and, more generally, the following lemma is easy to prove.

Lemma 1. In a tree T which is not a path, there is one vertex v for which the number of branch paths at v is $\deg(v)$, or there are at least two vertices, v_1 and v_2 , for which the number of branch paths at v_1 and v_2 is $\deg(v_1) - 1$ and $\deg(v_2) - 1$, respectively.

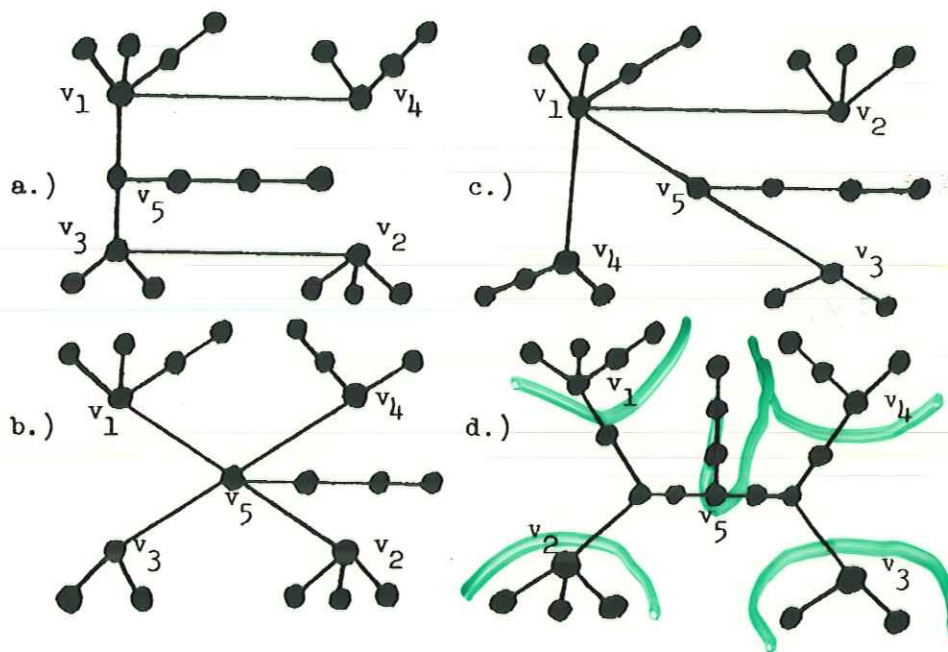
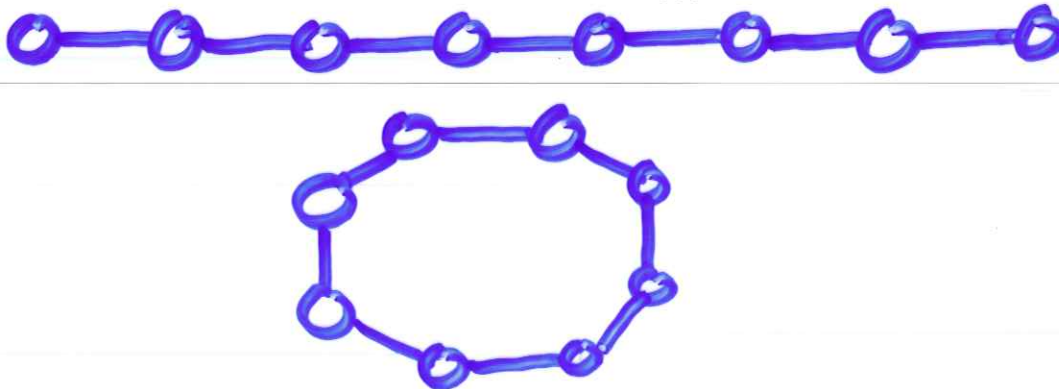


Figure 1. Four trees with the same leaf structure.



$$RED: LOC(G) \leq LOC(G) [1 + 2 \cdot 5^{LOC(G)-1}]$$



$$RED: \gamma(G) / \delta(G) \rightarrow \infty$$

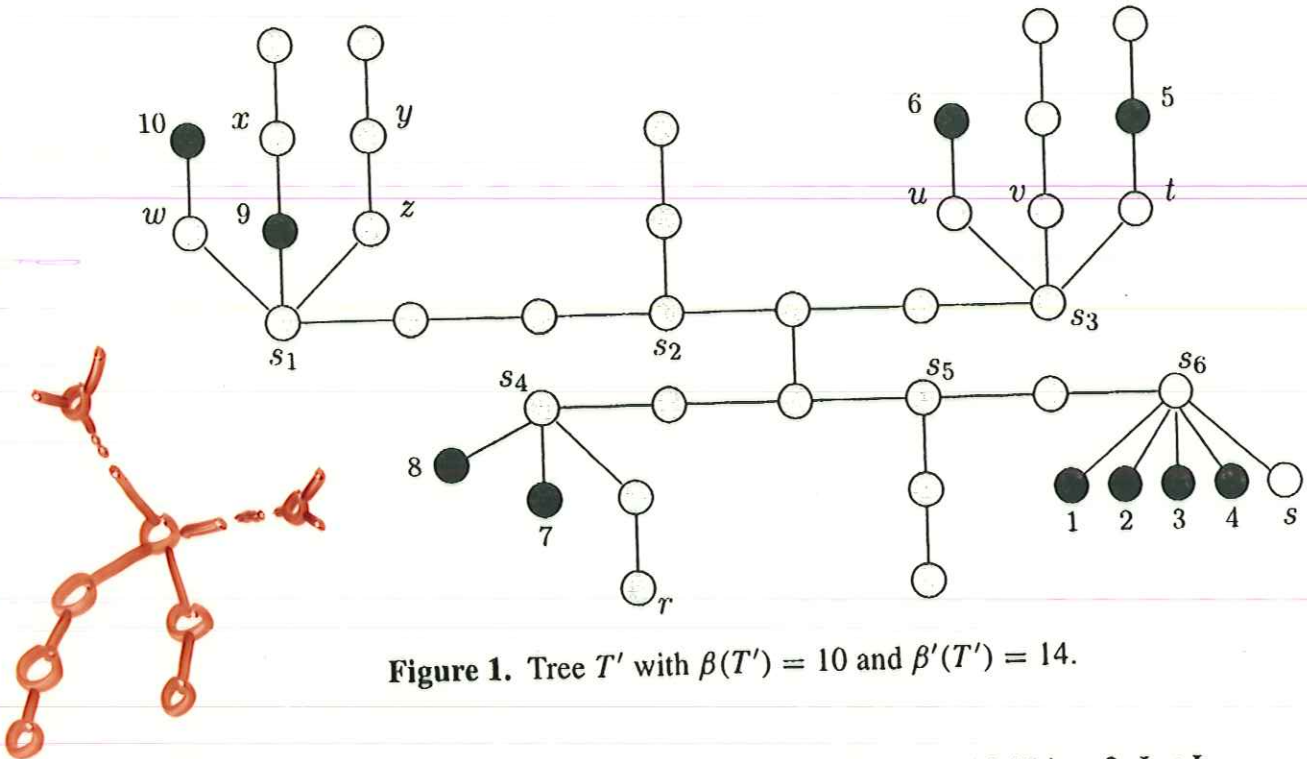


Figure 1. Tree T' with $\beta(T') = 10$ and $\beta'(T') = 14$.

Theorem 2.2 ([6]). Let T be a tree with set L of endpoints with $|L| \geq 3$. Let L_1, \dots, L_k be the components of the subtree induced by the set of all branch paths, and let e_i be the number of branch paths in T that are in L_i . Then $\beta(T) = |L| - k$, and S is a metric basis if and only if it consists of exactly one vertex from each of exactly $e_i - 1$ of the branch paths of L_i , for each L_i , $1 \leq i \leq k$.

Assume that S is a fault-tolerant resolving set for tree T . For each v in S , the set $S \setminus \{v\}$ is a resolving set. By Theorem 2.1, for any $v \in S \cap L_i$, the set $S \setminus \{v\}$ must contain a vertex from each of $e_i - 1$ of the branch paths of L_i . Thus, $|S \cap L_i| \geq e_i$ when $e_i \geq 2$. If L is the set of endpoints and E_1 is the set of endpoints to branch paths where $e_i = 1$, then, by Theorem 2.1, $L \setminus E_1$ is a fault-tolerant resolving set. This implies the next theorem.

Theorem 2.3. Let T be a tree with set L of endpoints with $|L| \geq 3$. Let L_1, \dots, L_k be the components of the subtree induced by the set of all branch paths, and let e_i be the number of branch paths in T that are in L_i . Let E_1 be the set of endpoints corresponding to branch paths where $e_i = 1$. Then $\beta'(T) = |L \setminus E_1|$ and $L \setminus E_1$ is a fault-tolerant metric basis.

Remark 2.4. If S is a fault-tolerant metric basis for T and s_i is the stem vertex of L_i , then $e_i \geq 3$ implies that $S \cap L_i$ will consist of exactly one vertex from each path of $L_i \setminus \{s_i\}$, and, if $e_i = 2$, then $S \cap L_i$ can be any two vertices of $L_i \setminus \{s_i\}$.

In particular, for the tree T' of Figure 1, $\beta'(T') = 14$ and $(1, 2, 3, \dots, 10, y, v, r, s)$

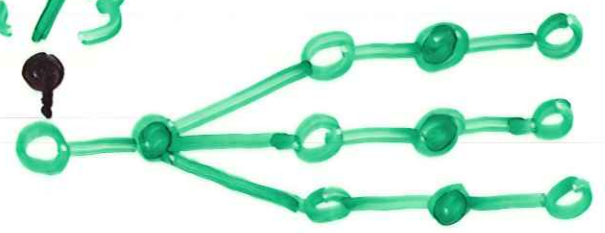
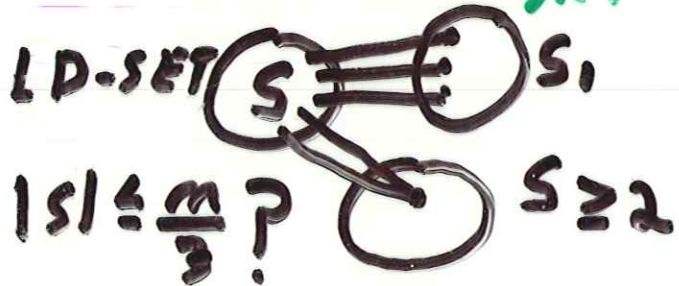
LOCATING-DOM. $S \subseteq V(G)$

$\forall u, v \in S, N(u) \cap S \neq N(v) \cap S \neq \emptyset$

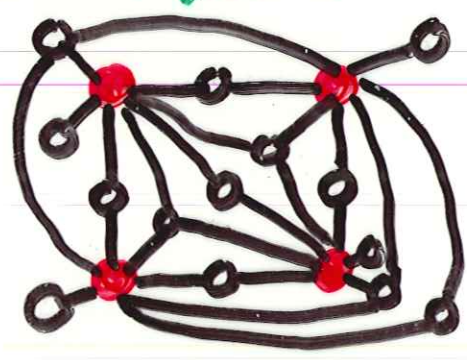


$\lfloor \frac{2}{3}n \rfloor = LD(P_n) = LD(C_n)$

THM. $LD(T_n) > n/3$



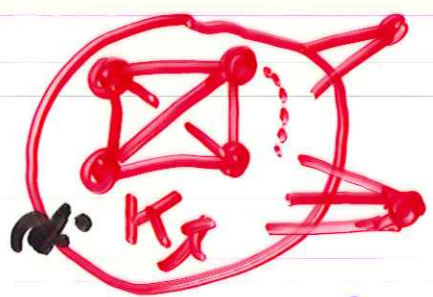
$n \leq 3LD(T_n) - 1$



$n \leq LD(G_n) + 2$
 (RALL, PS [LD-3])

$n \leq 7LD(P_n) - 10$

$n \leq 7LD - 10 + \log_2 ?$



$\beta = |V(G_j) - V(K_n)|$
 $= 2^x - 1$
 $i = 2^x - 1$

$i(K_n) = 1 < n-1 = LD(K_n)$
 (CHELLALI, MANOUNI, PS [LD-7])

$i(T) \leq LD(T)$

OTHER CLASSES ?

ZDZ

S DOM. SHARE $\gamma(n; S) = \sum_{w \in N[n]} \frac{1}{|S \cap N[w]|}$



S LD $1 + 1 + \frac{1}{2}(n-1)$ $LD(G_n) \geq \frac{2}{n+3} n$
 $\Rightarrow sl(w) \leq \frac{n+3}{2}$ $LD_{90}(G_n) \geq \frac{2}{n+3}$
 $\frac{2}{7} \leq LP_{90}(\mathbb{Z} \times \mathbb{Z}) = 3/10$

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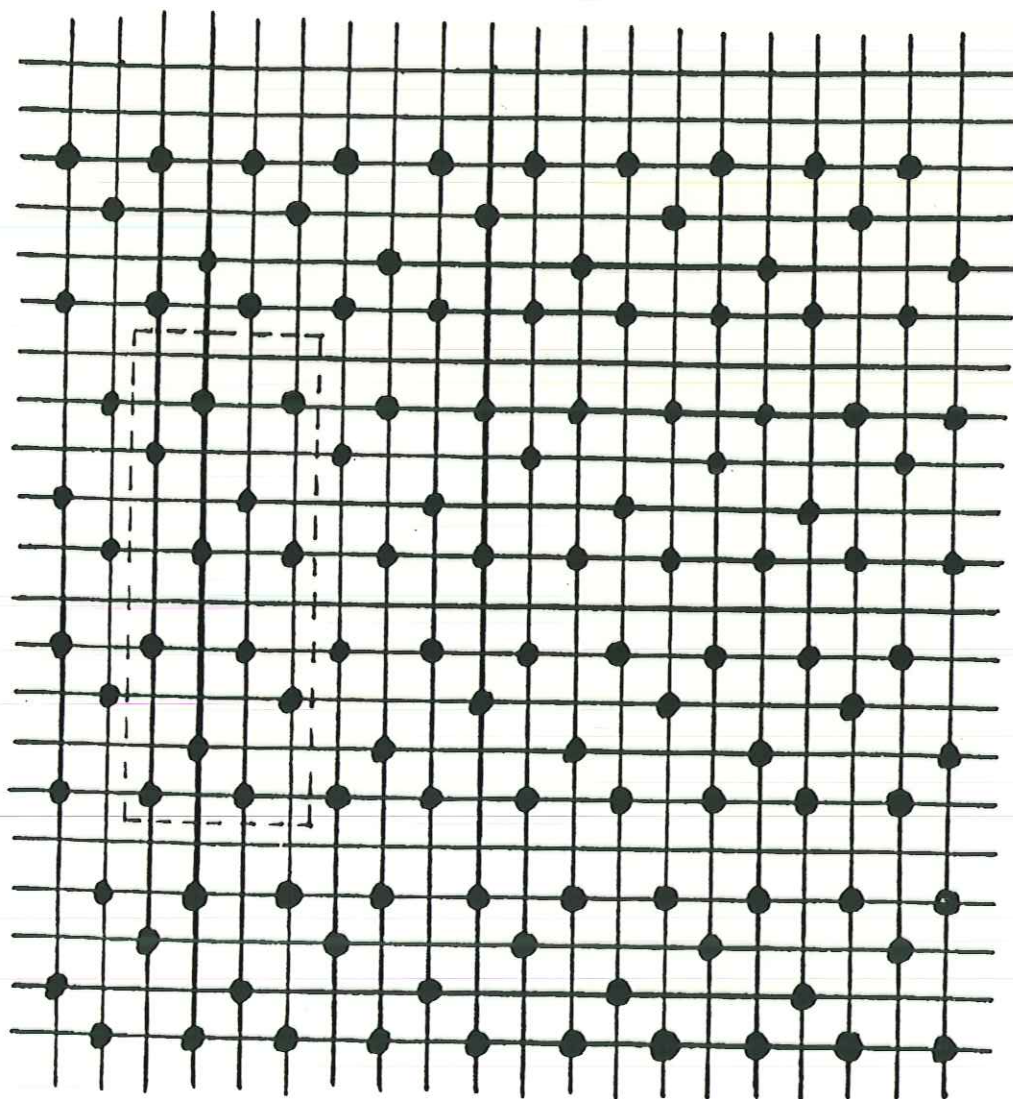
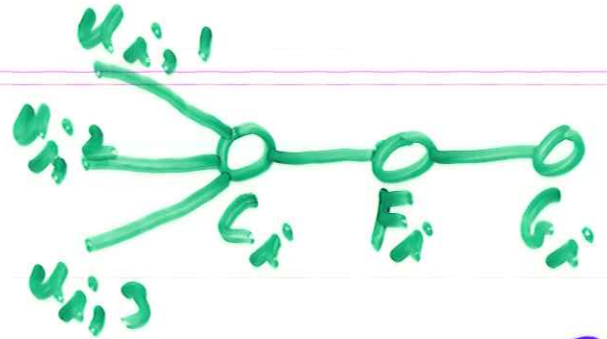
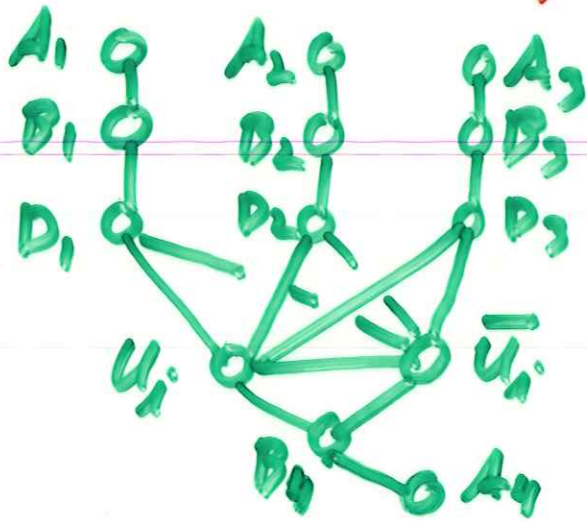


Fig. 2. An optimal locating-dominating set for $\mathbb{Z} \times \mathbb{Z}$.

and all of the values of $1/6$ shifted to y from vertices x (including $x = v$) with $sl(\gamma(x, S)) = 7/2$ will be a total of at most $10/3$

3SAT $U = \{u_1, \dots, u_N\}$

$C = \{c_1, c_2, \dots, c_M\}$ $|c_i| = 3$

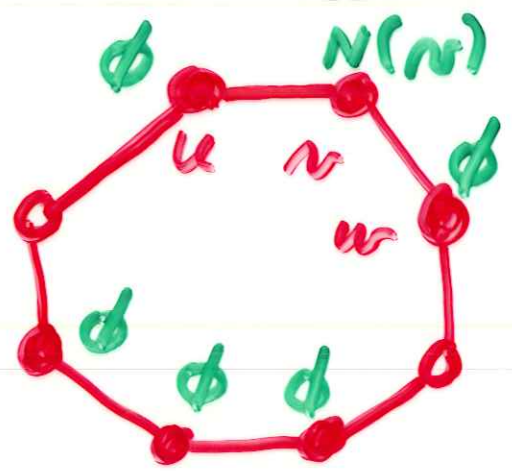


$LD(G) \leq 5N + M?$

FTLD

RED: ≤ 1 FAILURE TO TRANSMIT

FT: ≤ 1 (UNKNOWN WHICH) FAILURE TO DETECT, TRANSMIT ϕ (ANY $\{v\}$ OR $N(v)$ IS TRUE)



THM. $RED:LD(G) \leq FT:LD(G)$

THM. \exists a FTLD(G)-SET S iff $N(u) \neq N(v)$

THM. $FTLD(T_n) > (4/5)n$

THM. $v \in FTLD\text{-set } S \Rightarrow |\{w \in N(v) : |N[w] \cap S| = 2\}|$

Cor. $f(v; S) \leq \frac{1}{2} + \frac{1}{2} + \frac{1}{3}(\deg v - 1) \leq \frac{1}{3}$
 $= (2 + \deg v) / 3$

Cor. G r -reg. $FTLD(G_n) \geq \frac{3}{2+r} \cdot n$

$FTLD\%(G^\infty) \geq 3/(2+r)$

Cor. $FTLD\%(Z_2) \geq \frac{3}{6} = 1/2$.

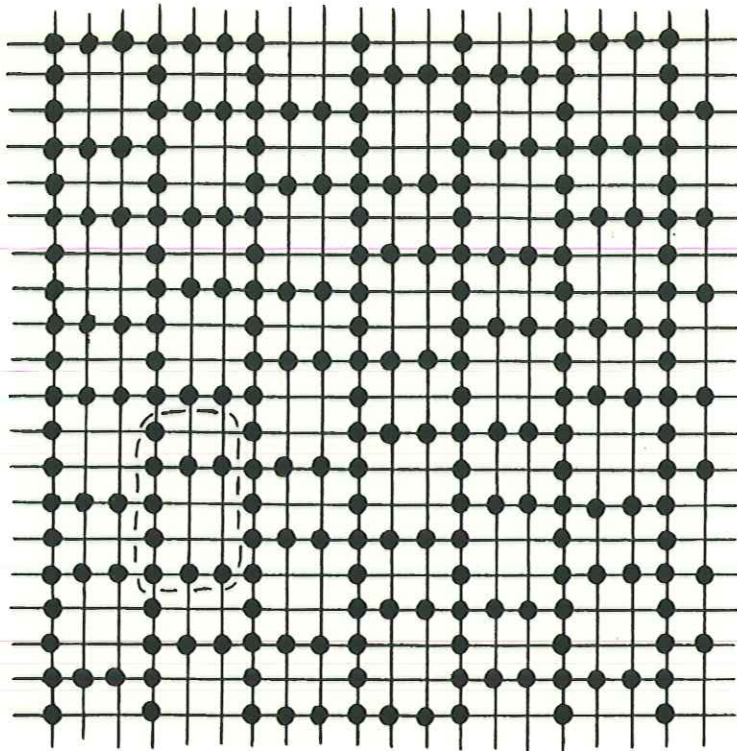


Fig. 3. $FTLD\%(Z \times Z) \leq 0.60$.

Theorem 10. $12/23 \leq FTLD\%(Z \times Z) \leq 3/5$.

Proof. The pattern illustrated in Fig. 3 shows a tiling of $Z \times Z$ using nine vertices in S in each 5-by-3 block, and darkened vertices form an FTLD-set for $Z \times Z$, so $FTLD\%(Z \times Z) \leq 9/15$.

Let S be an FTLD-set for $Z \times Z$ and $v \in S$. As noted, $\gamma(v, S) \leq 1/2 + 1/2 + 1/3 + 1/3 + 1/3 = 2$. It will be shown that $\gamma(v; S) = 2$ is not possible for any $v \in S$. Assume $v = (i, j)$ and $\gamma(v; S) = 2$. We have $|N[v] \cap S| = 2$ and can assume $(i-1, j) \in S$. Exactly one element $v^* \in N(v)$ contributes $1/2$ to $\gamma(v; S)$, and each of the other three contributes $1/3$.

Assume $v^* = (i-1, j)$. Then $S \cap \{(i-2, j), (i-1, j-1), (i-1, j+1), (i, j-1), (i, j+1), (i+1, j)\} = \emptyset$. Each of $(i, j-1)$, $(i, j+1)$ and $(i+1, j)$ have exactly three neighbors in S , so $\{(i, j-2), (i, j+2), (i+1, j-1), (i+1, j+1)\} \subseteq S$ and $(i+2, j) \notin S$. But now an intruder at $(i+1, j)$ and $(i+1, j-1)$ faulty and an intruder at $(i, j+1)$ and $(i, j+2)$ faulty would both result in (i, j) and $(i+1, j+1)$ reporting a 1 and everything else reporting 0, a contradiction.

Assume $v^* = (i+1, j)$, so $|N[v^*] \cap S| = 2$ and $|N[(i, j-1)] \cap S| = |N[(i, j+1)] \cap S| = |N[(i-1, j)] \cap S| = 3$. If $(i+2, j) \in S$, then $(i+1, j-1) \notin S$, $(i+1, j+1) \notin S$, $(i, j-1)$

LIAR'S LIARS'

DOMINATION γ

INTRUDER VERTEX $x \in V(G)$

$$\gamma_{LR}(G) \equiv LR(G) \equiv LR:\gamma(G)$$

$$\equiv \gamma_{LR(1,1)}(G)$$

LR: γ

LR: γ -SET $L \subseteq V(G)$: $\hookrightarrow \gamma_{LR(i,j)}(G)$

FOR $v \in L$, $x \notin N[v] \Rightarrow v \mapsto \emptyset$

ONE $w \in N[x] \cap L$ CAN HAVE

$w \mapsto \emptyset$ OR $w \mapsto y \in N[w]$

ELSE $v \in N[x] \cap L \rightarrow v \mapsto x$

$$\bullet \gamma_{x(2)}(G) \leq \gamma_{LR}(G) \leq \gamma_{x(3)}(G)$$

$u \mapsto s$ and $v \mapsto s \Rightarrow s = x$

$u \mapsto s, v \mapsto t \neq s, w \in L - \{u, v\}$

$\Rightarrow w$ TRUTHFUL

THM $L \subseteq V(G)$ is LR-DOM iff

(1) L is $\gamma_{x(2)}$ -DOM

(2) $\forall u \neq v, |L \cap (N[u] \cup N[v])| \geq 3$



COR EACH CNPT C OF $\langle L \rangle$ HAS $|C| \geq 3$.

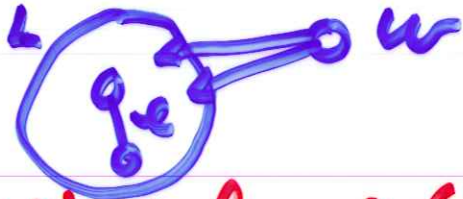
THM. $\Delta(G) = r \Rightarrow LR(G) \geq \frac{6}{3r+2} \cdot n$

R. (MINIMUM DEGREE ≥ 3 and $\delta_{v(2)}$ SUFFICES)

ASSUME LR(G)-set L , $|L| = k$

$\sum_{v \in L} \deg v \leq kr$

$v \in \langle L \rangle$ CONTRIBUTES 2 TO \sum

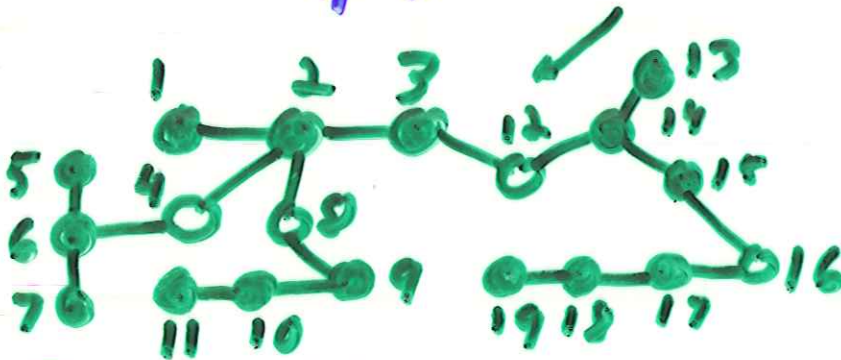


SO $|E(L, V(G) - L)| \leq kr - 2 \left(\frac{2}{3}k\right)$

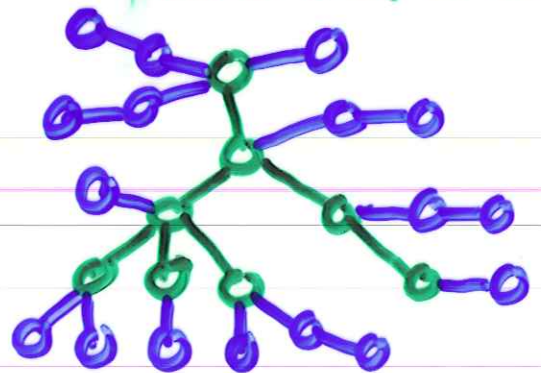
$\Rightarrow n \leq k + \frac{1}{2} [kr - \frac{4}{3}k]$

$= k \left(\frac{3r+2}{6}\right) \Rightarrow k \geq \frac{6}{3r+2} \cdot n$

THM. $\frac{3}{4}(n+1) \leq LR(T_n) \leq n$



v : ENDPOINT
 $T-v$ HAS SOME $|C| \leq 2$



$$\gamma_{LR}(G) \equiv \gamma_{X(2,3)}(G)$$

SET-SIZED DOMINATION

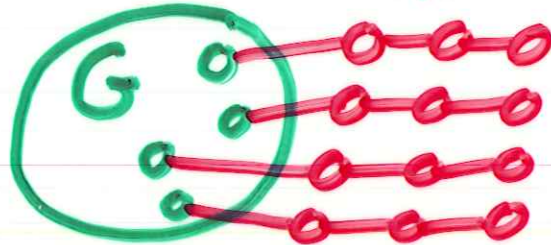
$$\gamma_X(c_1, c_2, c_3, \dots, c_x, \dots)$$

LRDOM: $\exists \gamma_{X(2,3)}(G) \leq J?$

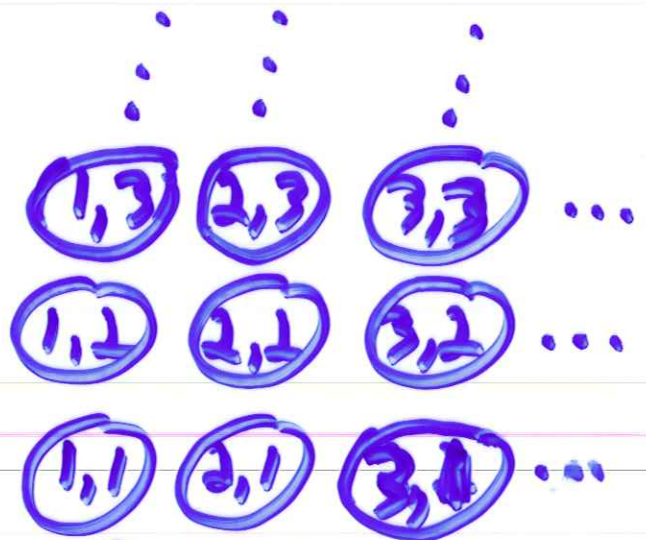
DOM: $\exists \gamma(G) \leq K?$

DOM \prec LR: $\gamma \equiv \gamma_{X(2,3)}$

$\gamma_{X(c_1, c_2)}(G) \leq J?$
is NP-c.



$\gamma_X(i, j)$



LR(i, j)

$$LR(1, j) \equiv \gamma_X(j+1, 2j+1)$$

$$LR(2, 2) \equiv \gamma_X(c_1, c_2, \dots)$$

$\mathcal{S} = \{S_1, S_2, \dots, S_k\}$ IS
DISTINGUISHING FOR G IF

- $\cup S_i = V(G)$
- $\forall u \neq v \exists S_i \ni |S_i \cap \{u, v\}| = 1$

$N[v_i] \neq N[v_j]$ IC $\{N[v_1], N[v_2], \dots, N[v_n]\}$

$N(v_i) \neq N(v_j)$ OLD $\{N(v_1), N(v_2), \dots, N(v_n)\}$

LOC $\{\underbrace{v_1, N(v_1), N^2(v_1), \dots,}$

$\underbrace{v_n, N(v_n), N^2(v_n), \dots}\}$

LD $\{\underbrace{v_1, N(v_1)}, \underbrace{v_2, N(v_2)}, \dots,$
 $\underbrace{v_n, N(v_n)}\}$

$\mathcal{S} \{ \underbrace{v_{1,1}, \dots, v_{1,1+d_1}}, v_{2,1},$

$\dots, v_{m,1}, \dots, v_{m,1+d_m}\}$

$\mathcal{S}(v) = \{i \mid v \in S_i\} \neq \emptyset$
 $| \mathcal{S}(u) \Delta \mathcal{S}(v) | \geq 1$

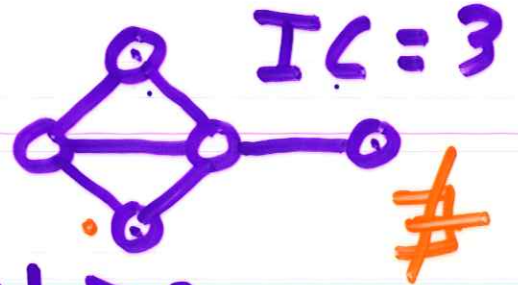
\mathcal{S} is REDUNDANT DISTING.
 FOR G if all $\mathcal{S}-\{S_i\}$ are DISTING.

\Rightarrow " $\mathcal{S}_{\times(2)}$ "

$\mathcal{S} \quad \delta(G) \geq 1$

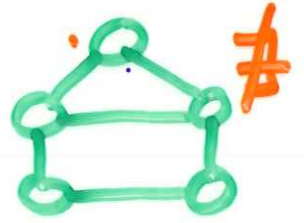
LD $\delta(G) \geq 1$

LOC $\delta(G) \geq 1$



IC $|N(u) \Delta N(w)| \geq 2$

OLD $|N(u) \Delta N(w)| \geq 2$ OLD=3



$\mathcal{S}: |S(u)| \geq 2 \quad |S(u) \Delta S(w)| \geq 2$

\mathcal{S} is FT-DISTINGUISHING if \mathcal{S}
 CAN DISTING. UNDER THE CONDITION
 THAT ONE S_i CAN FALSELY
 REPORT \emptyset .

FT:IC

$S \subseteq V(G)$

- S DOUBLE DOMINATES
- \forall PAIR $u, v \exists x, y \in S$
 $\exists \emptyset \neq N[x] \cap \{u, v\}$
 $= N[y] \cap \{u, v\} \neq \{u, v\}$

\mathcal{S} is FT-DISTING.:

- $\forall u$ WE HAVE $|\mathcal{S}(u)| \geq 2$
- $\forall u, v \exists i, j$ WITH
 $\emptyset \neq S_i \cap \{u, v\} = S_j \cap \{u, v\} \neq \{u, v\}$

EITHER $|\mathcal{S}(u) \setminus \mathcal{S}(v)| \geq 2$

OR $|\mathcal{S}(v) \setminus \mathcal{S}(u)| \geq 2$

$$\gamma \leq \text{RED}:\gamma \equiv \gamma_{x(2)} \quad \gamma \leq \text{LR}:\gamma \equiv \gamma_{x(2,3)} \quad \gamma \leq \text{ER}:\gamma \equiv \gamma_{x(3)}$$



$\gamma \quad 1/3$



RED: $\gamma \equiv \gamma_{x(2)} \quad 2/3$

FT: $\gamma \equiv \gamma_{x(2)} \quad 2/3$



LR: $\gamma \equiv \gamma_{x(2,3)} \quad 3/4$



ER: $\gamma \equiv 1$

IF $|S \cap N[w]| = |\{a, \emptyset\}|$
 AND $a \mapsto w, \emptyset \mapsto \emptyset$

THEN INTRUDER AT w
 OR \nexists AN INTRUDER

ER: $\gamma \equiv \gamma_{x(3)}$

ER⁻: γ

$$LD \leq RED:LD \leq FT:LD \leq LR:LD \leq ER:LD$$

$$Y_{x(2)} \leq$$



LD 2/5



RED:LD 2/3



FT:LD 4/5



LR:LD 4/5



ER:LD 1

$$LOC \leq RED:LOC \approx FT:LOC \leq LR:LOC = ER:LOC$$



$$FT:LOC \equiv RED:LOC$$



LR:LOC 4 LR:LOC $(P_m) = 4, m \geq 4$

FOR $m \geq 4,$

$$3 \leq LR:LOC(G_m) = ER:LOC(G_m)$$

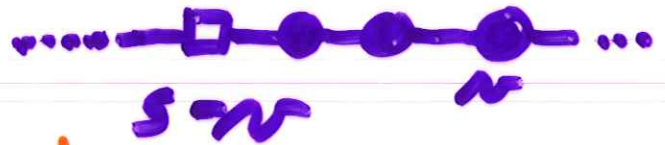
$$IC \leq RED:IC \leq FT:IC \leq LR:IC \leq ER:IC$$



IC $\frac{V}{2}$



RED:IC "1"



RED:IC(G) \nexists if $\exists w \exists$
 $|G| = w$ has a 2-COMPNT



IC = 2h

RED:IC = 3h = m



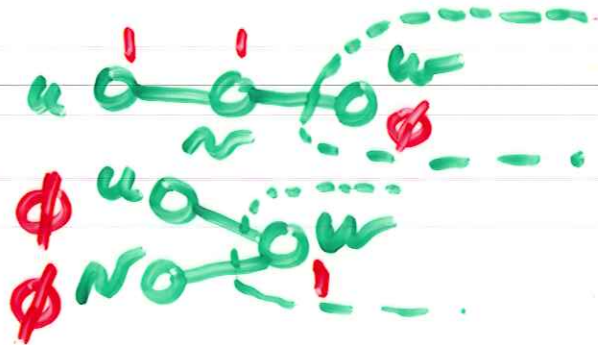
FT:IC

\nexists

0 0 1 1 0 0

u OR \sim

FT:IC(T) $\nexists \forall$ tree T



$$LR:IC \equiv FT:IC$$

DOMINATION LOC-DOM IC OLD LOC ...

REDUNDANCE: FAILURE TO TRANSMIT

RED:Y RED:LD RED:IC RED:OLD RED:LOC

FAULT TOLERANCE: UNKNOWN FAILURE
TO DETECT

FT:Y FT:LD FT:IC FT:OLD FT:LOC

LIAR'S: MISREPRESENTING
POSITIVE DETECTION

SET
SIZED
DOMINATION

LR:Y LR:LD LR:IC LR:OLD LR:LOC

ERROR: ANY MISREPRESENTING

ER:Y ER:LD ER:IC ER:OLD ER:LOC