



# LOCATING- AND DISTINGUISHING-TOTAL DOMINATION IN GRAPHS

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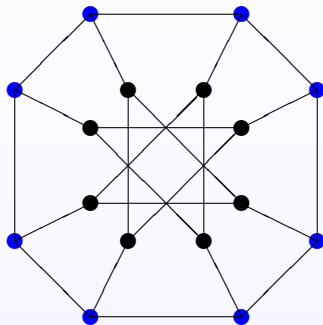
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- Total domination in graphs was introduced by **Cockayne, Dawes**, and **Hedetniemi** in **1980**. (Total domination in graphs. *Networks* **10** (1980), 211–219.)

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A graph  $G$  with  $\gamma_t(G) = 8$ .

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[9] **MAH** and **C. Löwenstein**, Locating-total domination in claw-free cubic graphs, manuscript (2011+).

[10] **MAH** and **J. McCoy**, Locating and paired-dominating sets in graphs. *Discrete Applied Math.* **157** (2009), 3268–3280.

[11] **MAH** and **J. McCoy**, Which trees have a differentiating-paired dominating set? *J. Combin. Optim.* **22** (2011), 1–18.

[12] **MAH** and **N. Rad**, Locating-total domination in graphs, manuscript (2011+).

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- Theorem 3 improves Theorem 2 for nontrivial trees with  $n > 4\ell + s - 6$ .

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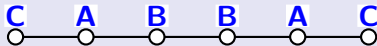
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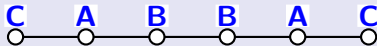
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- For  $k \geq 1$ , let  $T_k$  can be obtained recursively from  $T_{k-1}$  by one of the following operations.

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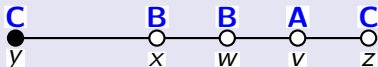
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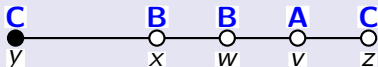
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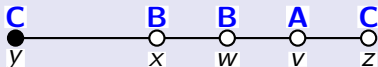


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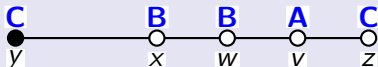
- Operation  $\tau_1$ . For any  $y \in V(T_{k-1})$ , if  $sta(y) = C$  and  $y$  is a leaf of  $T_{k-1}$ , then add a path  $xwvz$  and edge  $xy$ . Let  $sta(x) = sta(w) = B$ ,  $sta(v) = A$ , and  $sta(z) = C$ .



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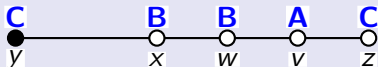
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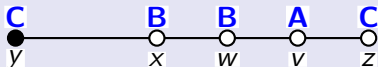
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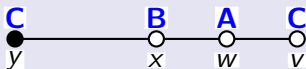
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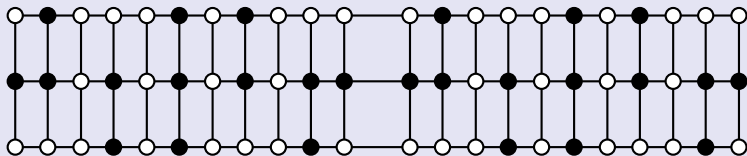
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A **LTD-set** for the grid  $P_3 \square P_{22}$

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- For example,  $\xi(4) = 3/2$  and  $\xi(6) = 2$ .

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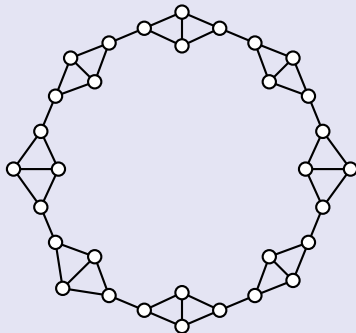
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A **diamond-necklace**  $N_8$  with eight diamonds.

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- What are the **extremal graphs**?



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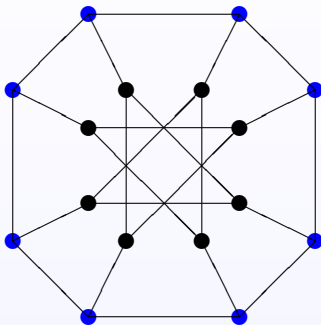
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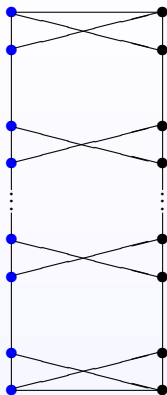
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# Sharpness of Theorem 12.



The **generalized Petersen graph**  $G_{16}$  with  $\gamma_t(G_{16}) = 8$ .

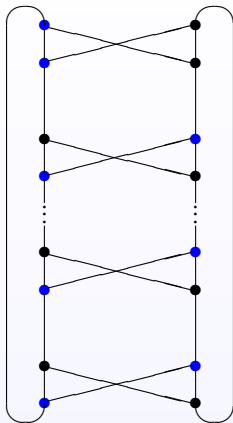
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- In general, the locating-total domination number of a cubic graph can be very much larger than its total domination number

## Locating-Total Domination in Cubic Graphs

- **Conjecture 3.**

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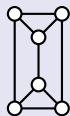
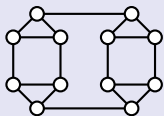
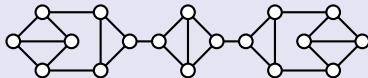
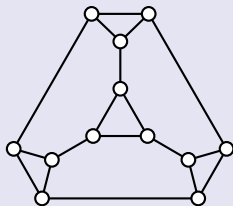
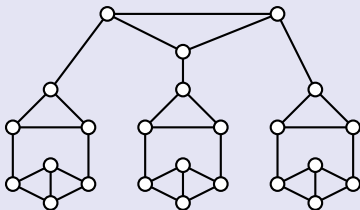
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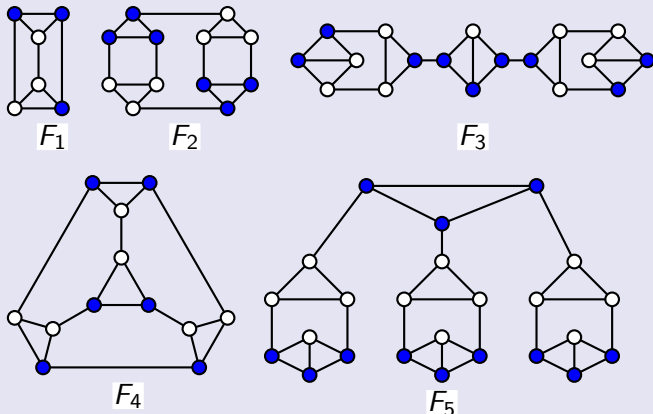
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## Locating-Total Domination in Claw-Free Cubic Graphs

 $F_1$  $F_2$  $F_3$  $F_4$  $F_5$ 

The family  $\mathcal{F}_{\text{cubic}} = \{F_1, F_2, F_3, F_4, F_5\}$ .

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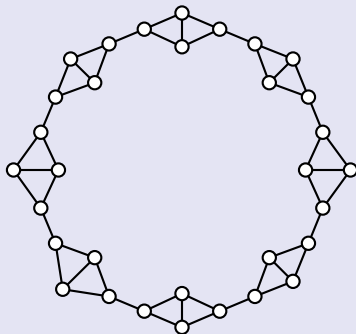
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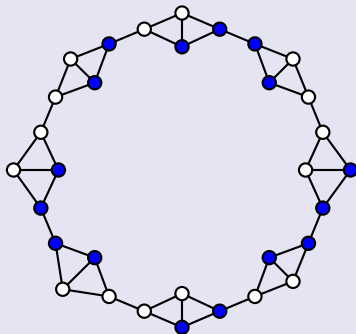
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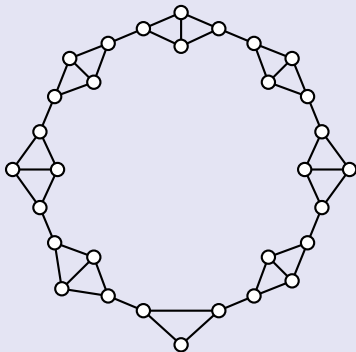
with equality if and only if  $\mathbf{G} \in \mathcal{N}_{\text{cubic}} \cup \mathcal{F}_{\text{cubic}}$ .

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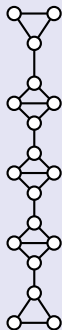
A diamond-bracelet  $B_7$ .

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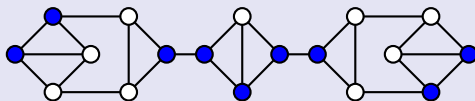
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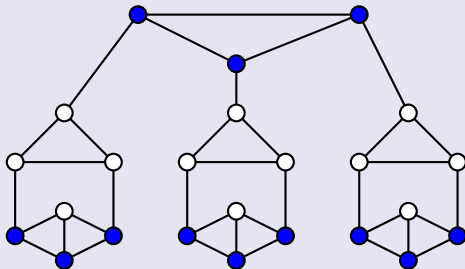
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- If  $\mathbf{G}$  contains a diamond-bracelet  $\mathbf{B}_k$ , where  $k \geq 1$ , then we show that either  $\mathbf{G} \in \{\mathbf{F}_3, \mathbf{F}_5\} \subset \mathcal{F}_{\text{cubic}}$  or  $\gamma_t^L(\mathbf{G}) < n/2$ .

## Locating-Total Domination in Claw-Free Cubic Graphs



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- For each **triangle-unit** in  $G$ , we associate a vertex of  $M$ .

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- Since  $G$  contains no diamond-bracelet,  $M$  is loop-free.

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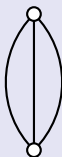
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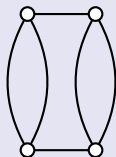


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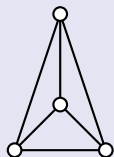
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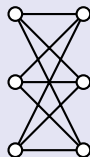
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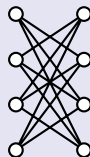
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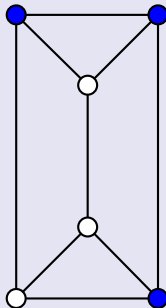
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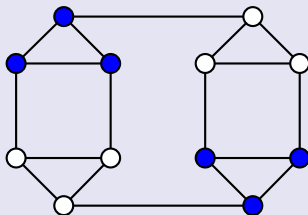


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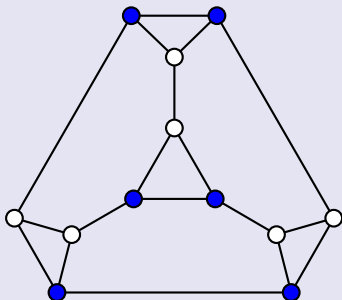
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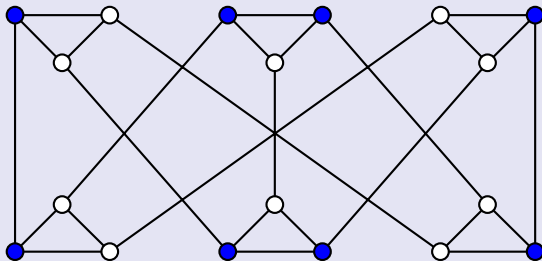
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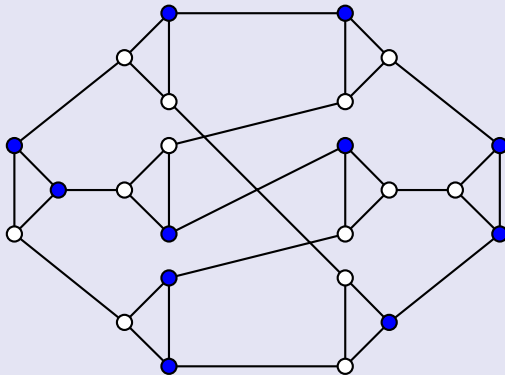
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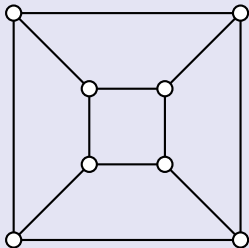
- Let  $G$  be a **connected cubic graph** of order  $n$  that is twin-free.
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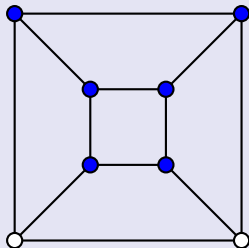
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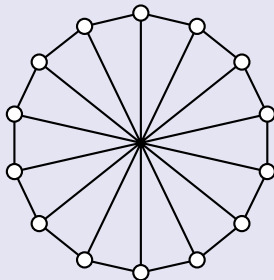
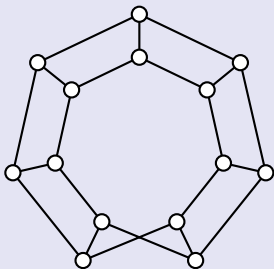
$$\gamma_t^D(Q_3) = 6 = \frac{3}{4}n$$

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- For an even number  $n \geq 4$ , the **Möbius ladder**  $M_n$  is a **cubic circulant graph** on  $n$  vertices, formed from an  $n$ -cycle by adding edges connecting opposite pairs of vertices in the cycle.

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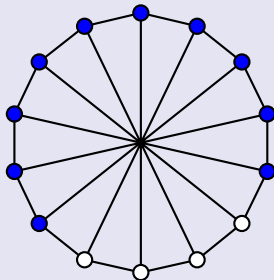
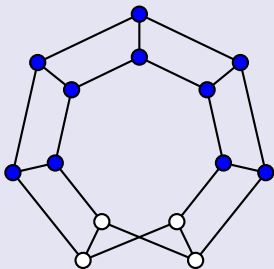
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Two views of the **Möbius ladder**  $M_{14}$ .

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$$\gamma_t^D(M_{14}) = 10 = \frac{5}{7}n$$

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- Let  $G$  be a **connected cubic graph** of order  $n$  that is twin-free.
- **Problem.** Find an upper bound on  $\gamma_t^D(G)$  in terms of  $n$ .

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- Every **(simple) graph** is a **2-uniform hypergraph**.

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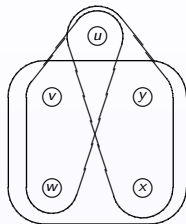
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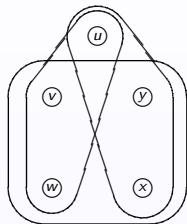
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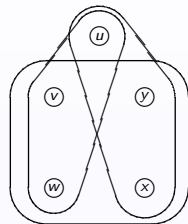
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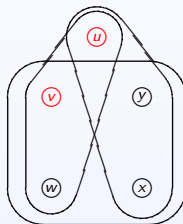
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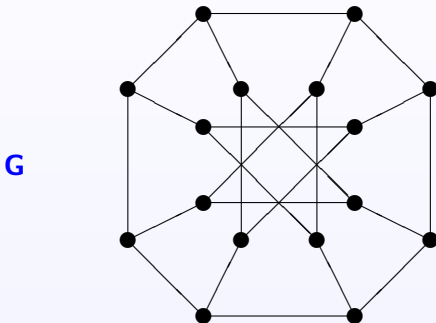
consisting of the **open neighborhoods** of vertices in  $G$ .

# Open Neighborhood Hypergraph

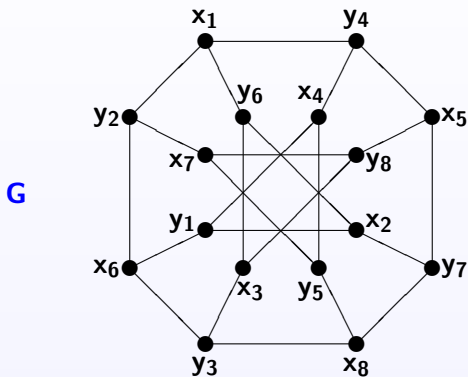
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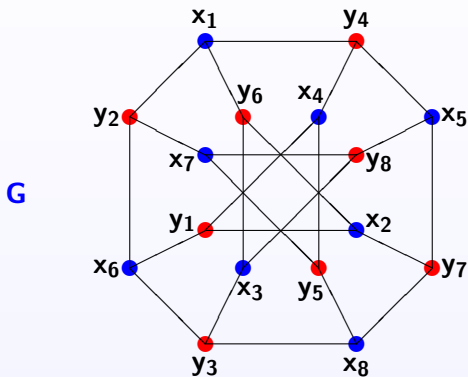
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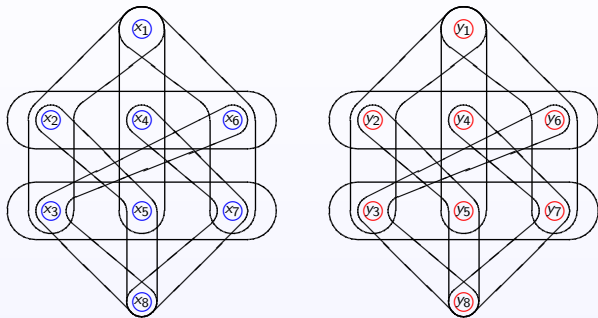
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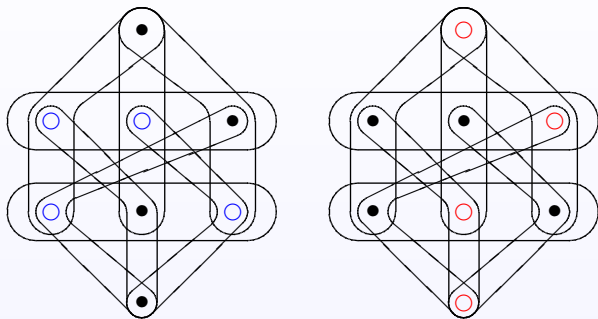


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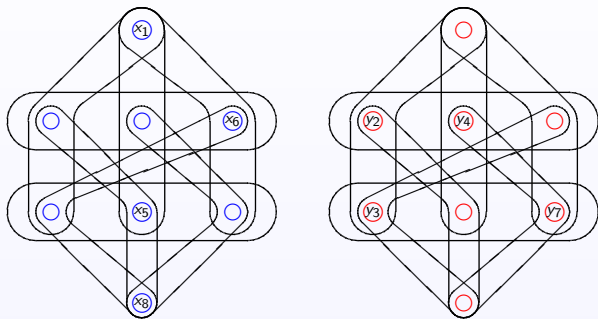
**ONH(G).**

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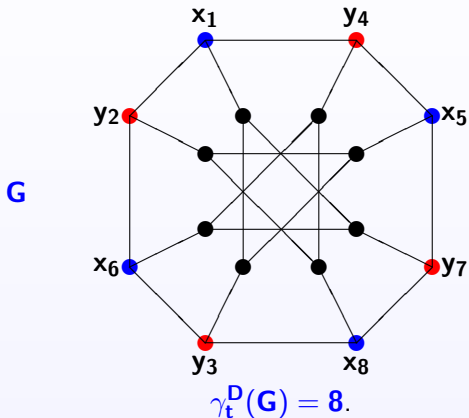
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For every graph  $G$  that is **twin-free**, we have

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- The main advantage of considering **hypergraphs** rather than **graphs** is that the **structure** is easier to handle.

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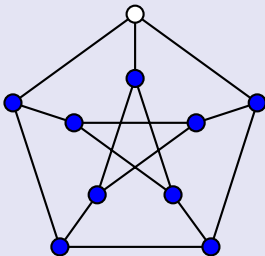
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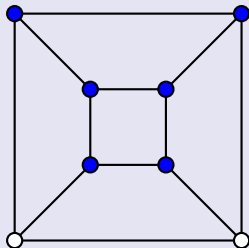
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## Distinguishing-Total Domination in Cubic Graphs

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- If **Conjecture 4** is true, then this bound would be sharp.

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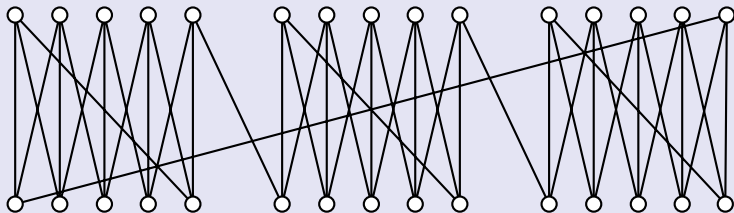
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- By construction, the graph  $G_k$  is a connected cubic graph that is twin-free.

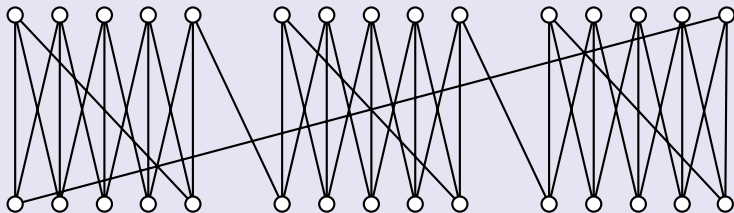
# Distinguishing-Total Domination in Cubic Graphs

## The hypergraph $G_3$



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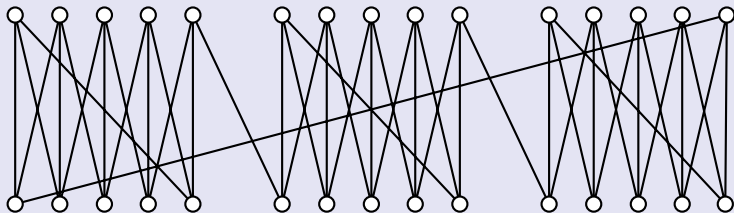
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**Proposition.** For  $k \geq 1$ , if  $G_k$  has order  $n$ , then  $\gamma_t^D(G_k) = \frac{3}{5}n$ .

**THANK YOU FOR YOUR ATTENTION!**

**NGIYABONGA KAKHULU!**