

# Counting resolving sets in graphs

**Antonio González**

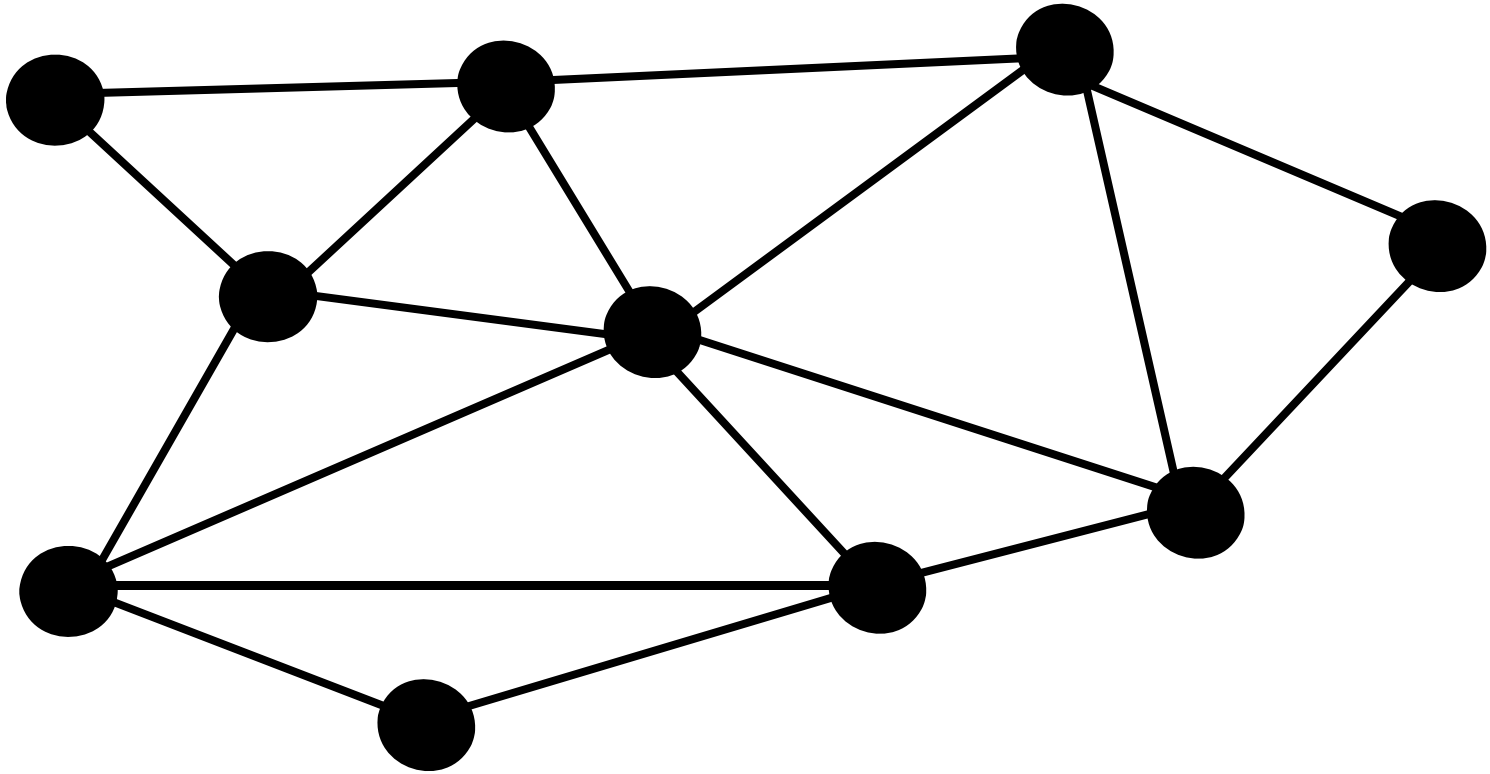
**University of Seville**

**Joint work with D. Garijo and A. Márquez**

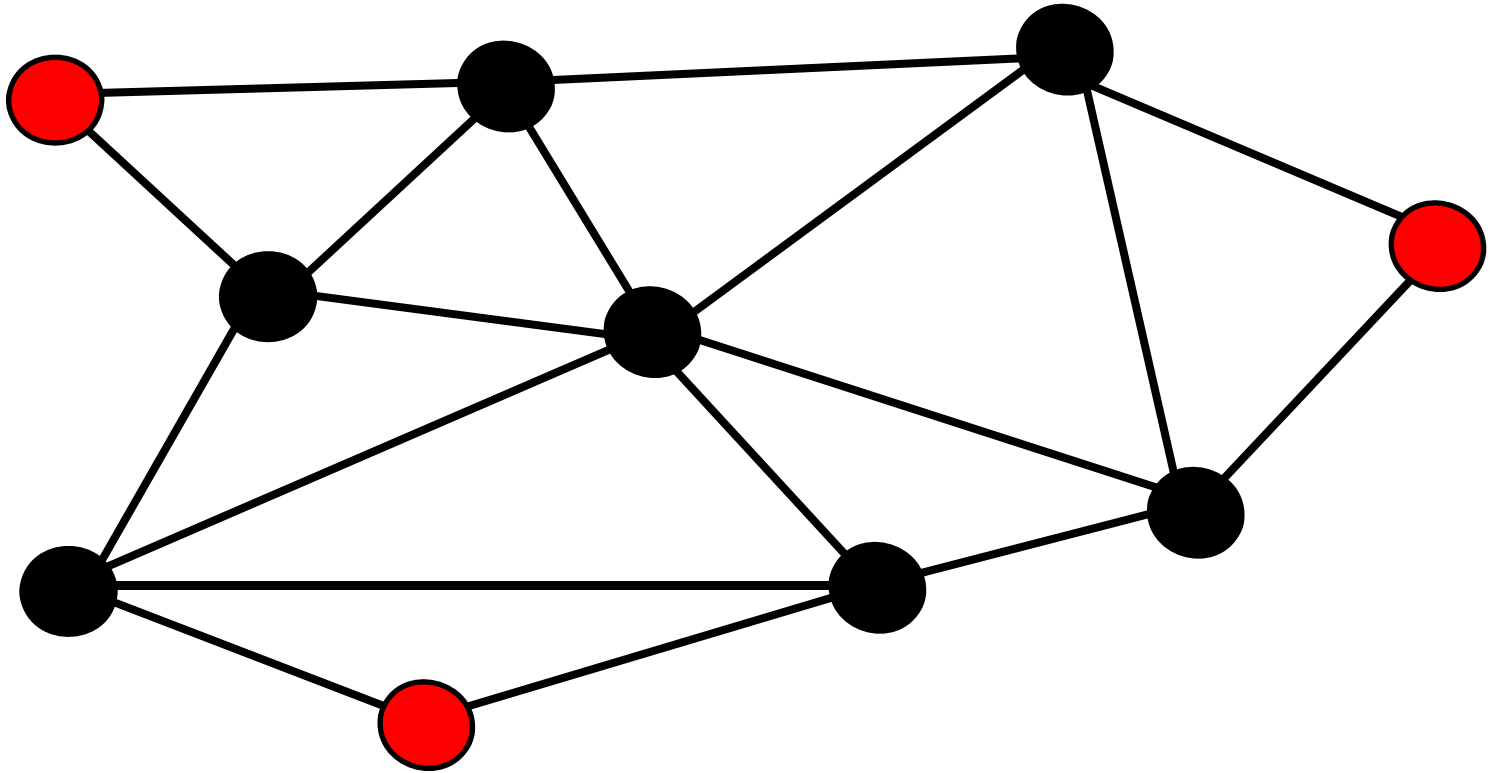


# Resolving Sets and Metric Dimension

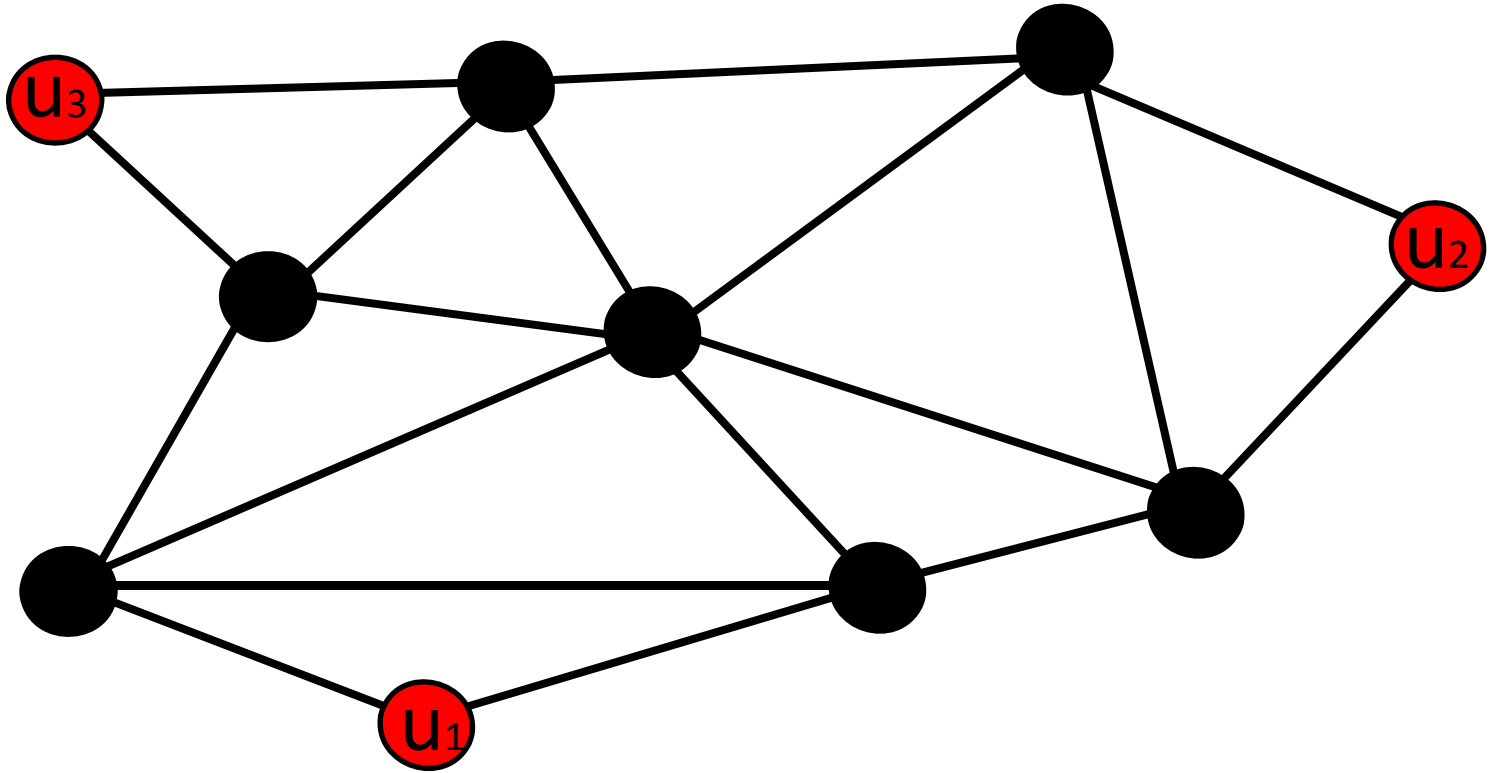
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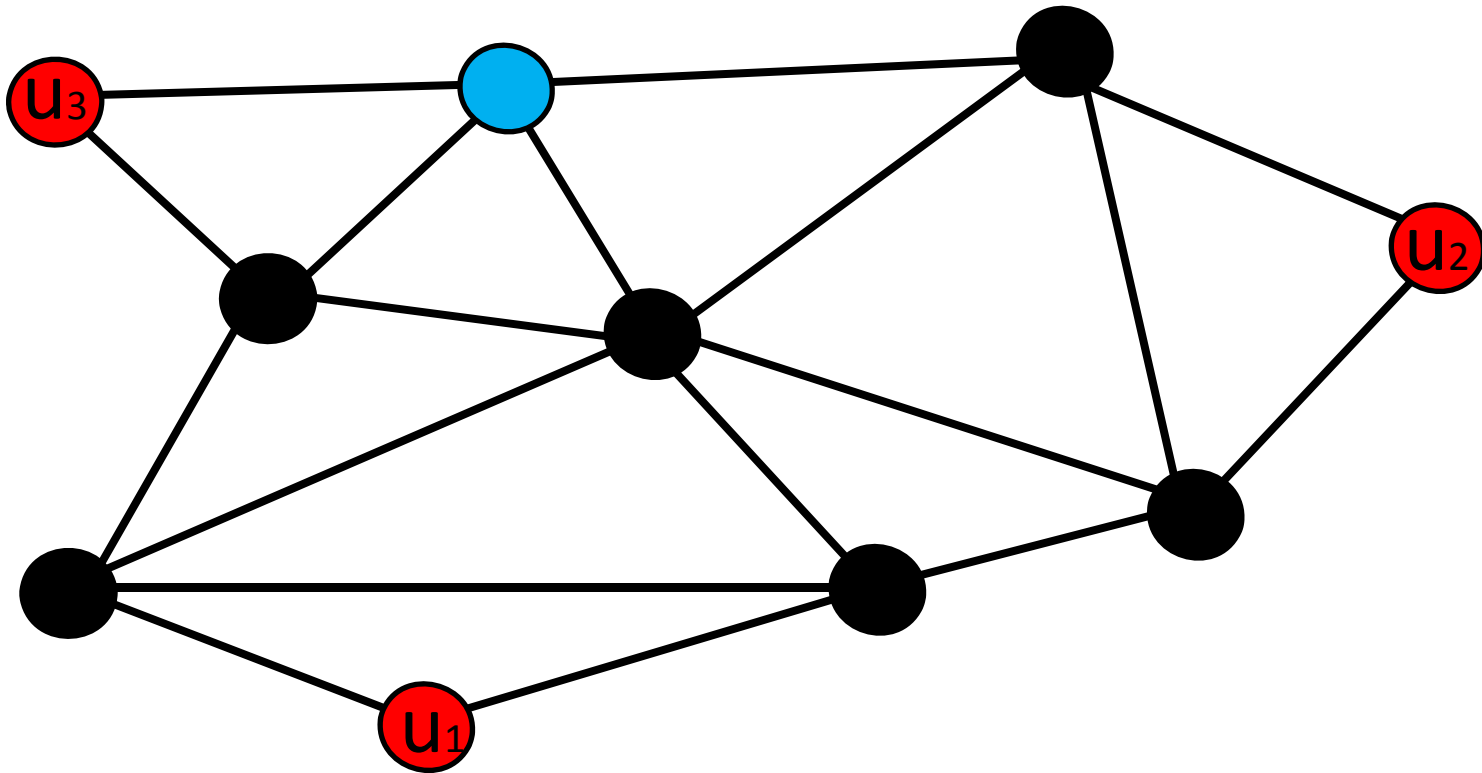
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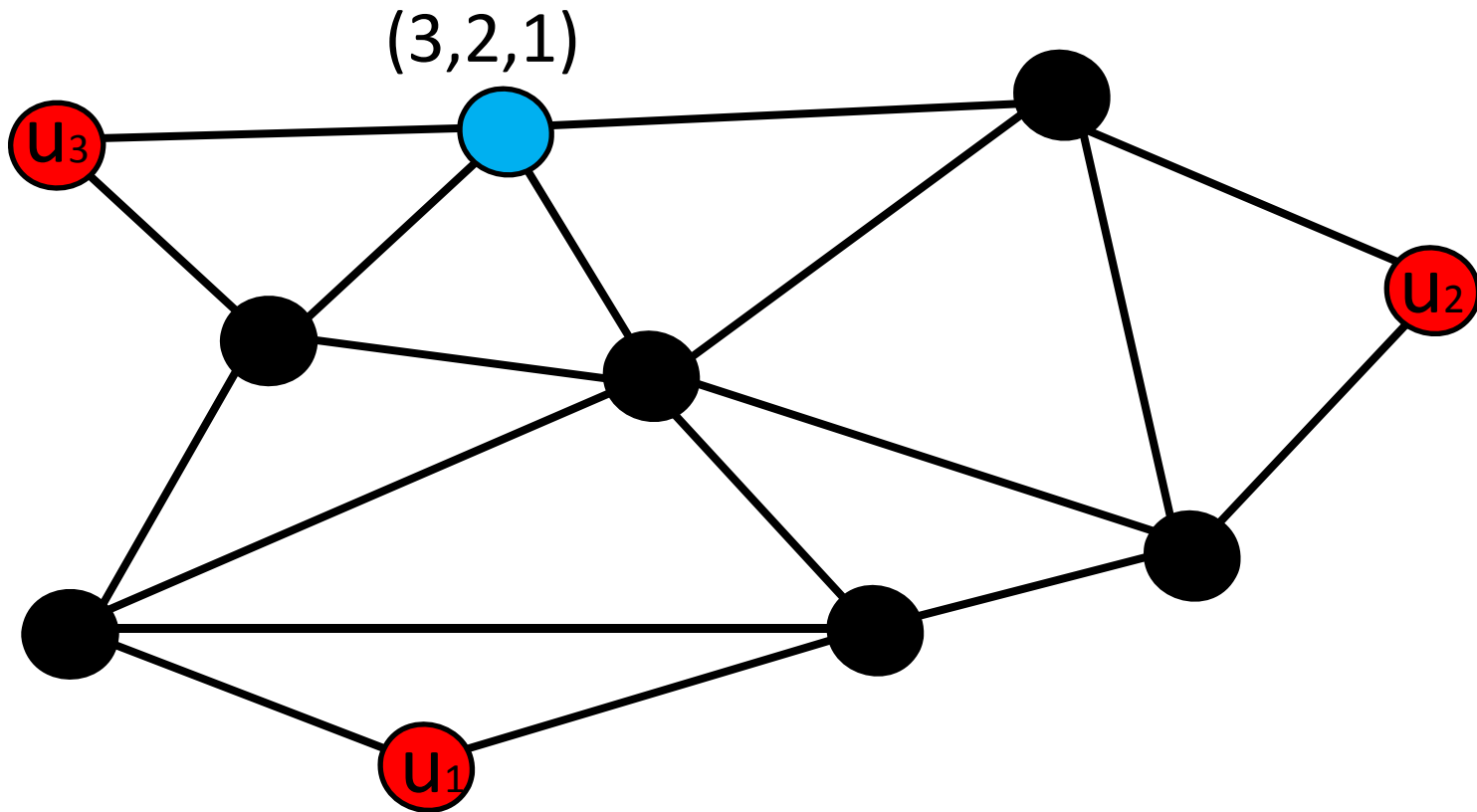
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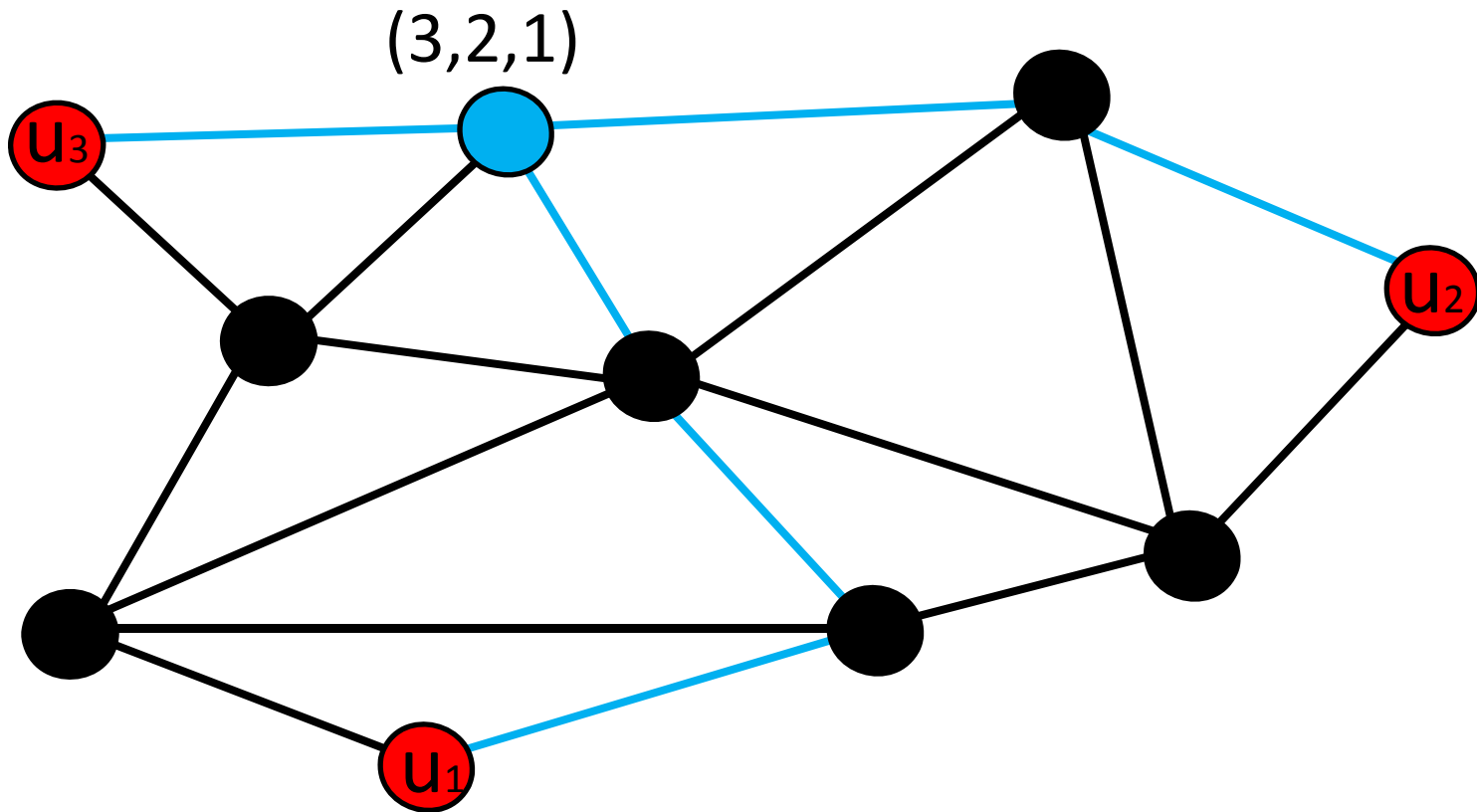


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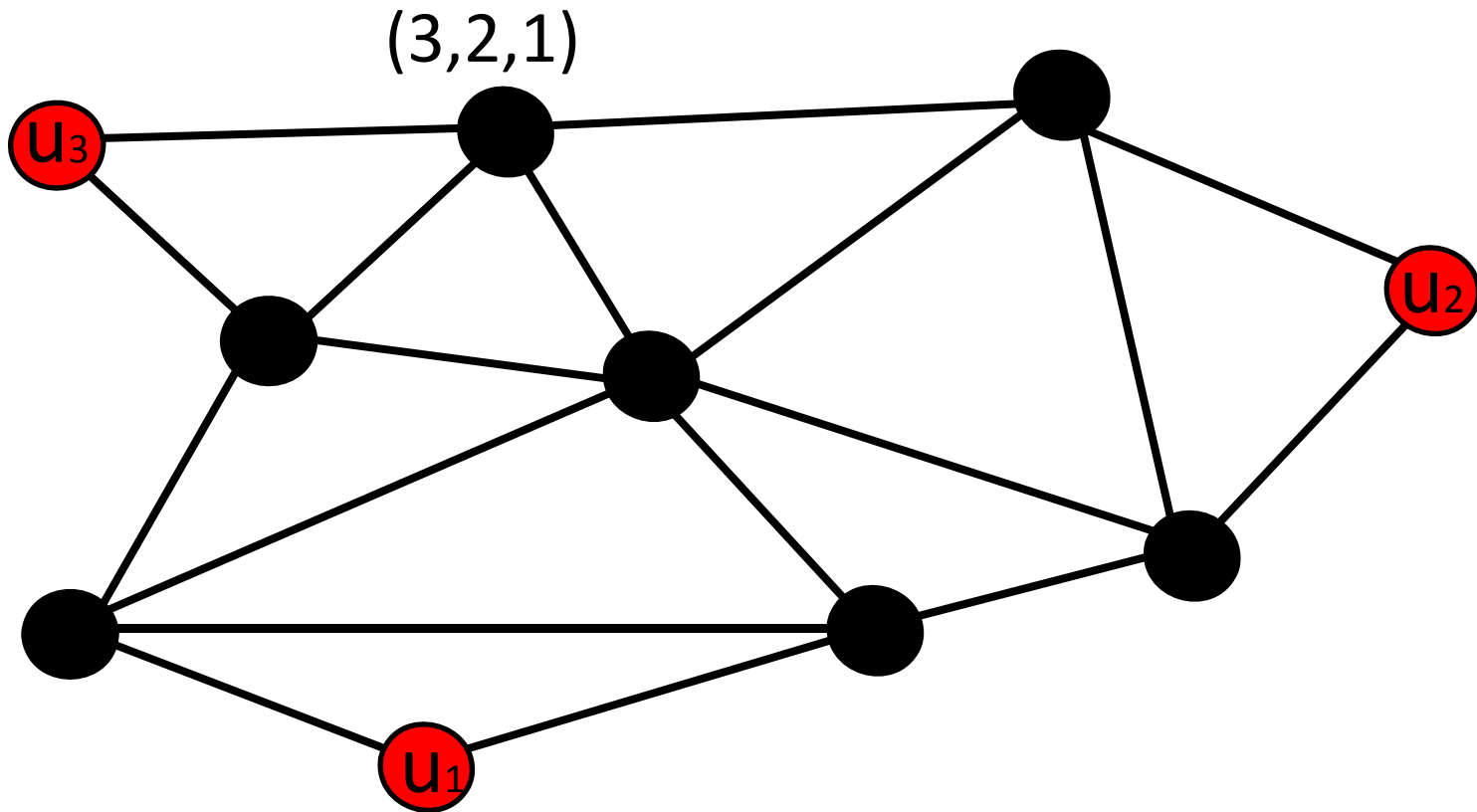




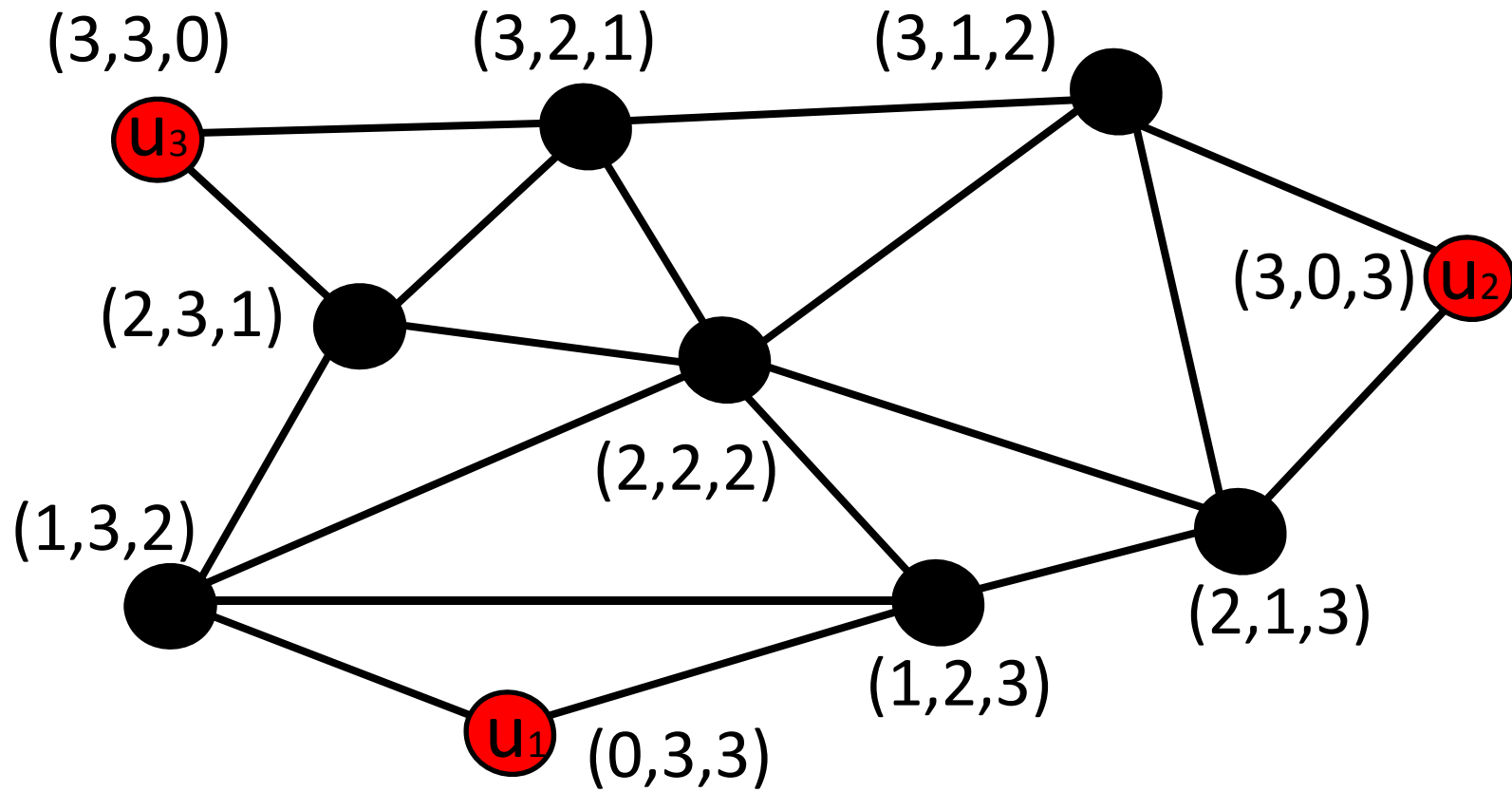
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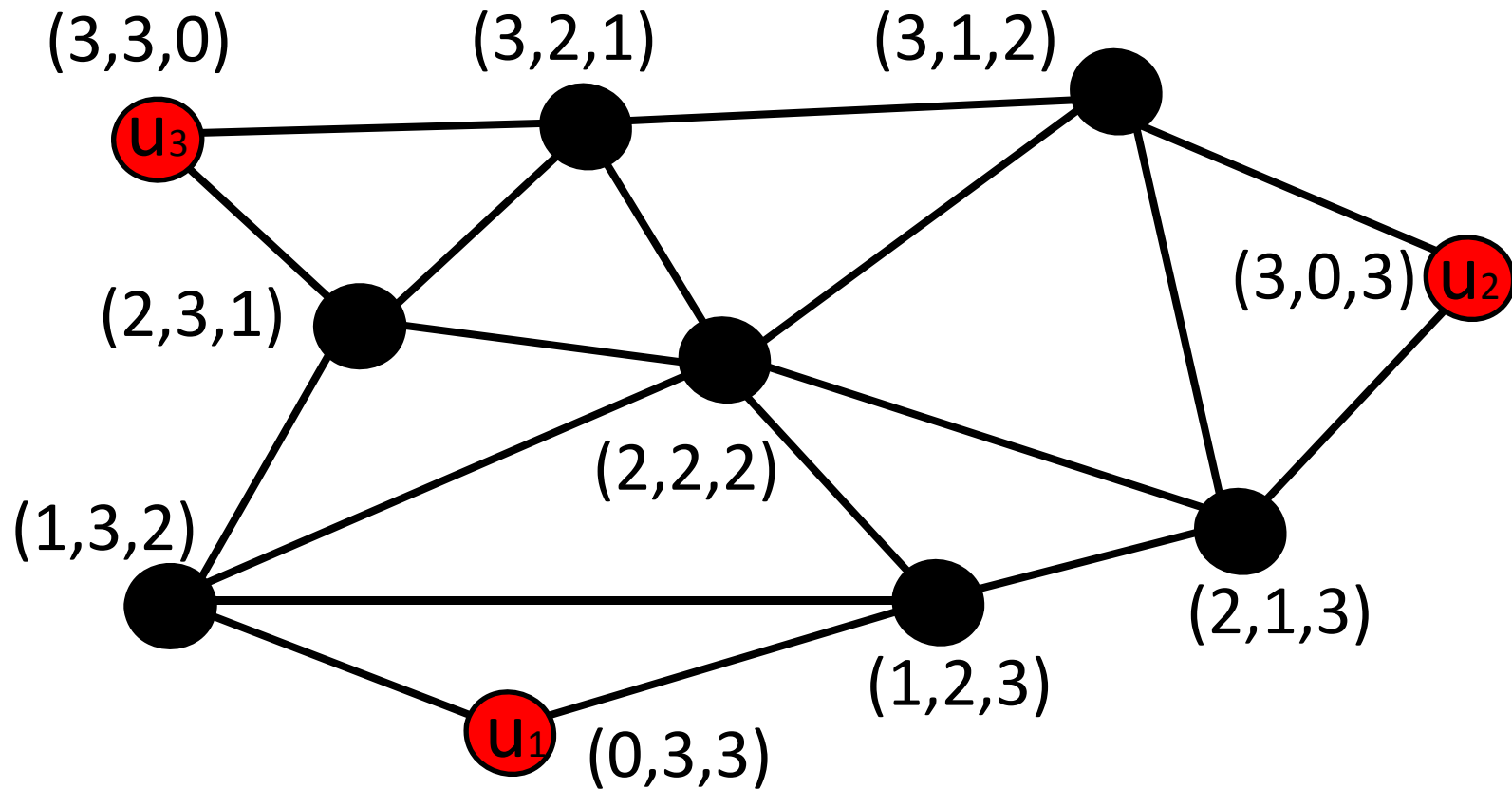
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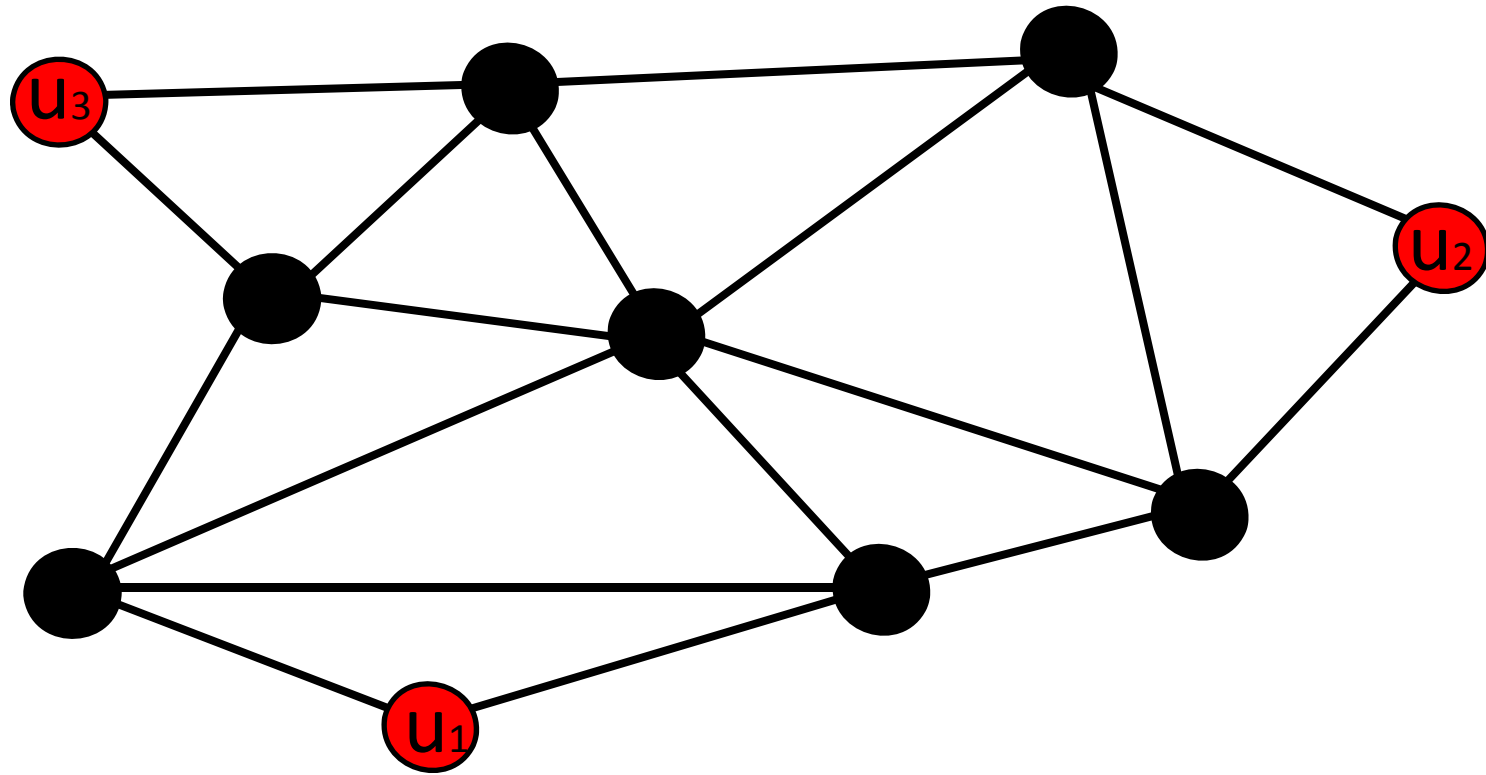
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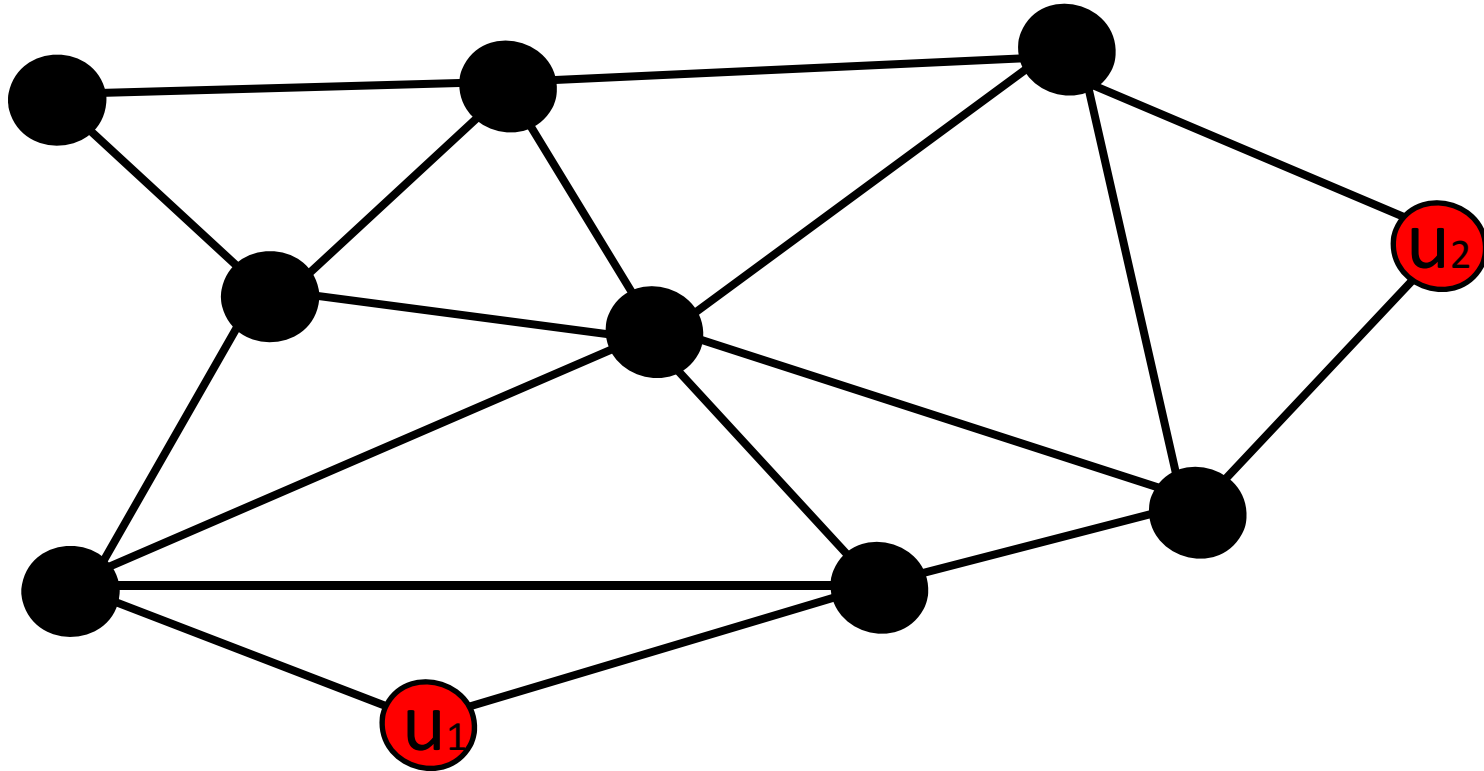
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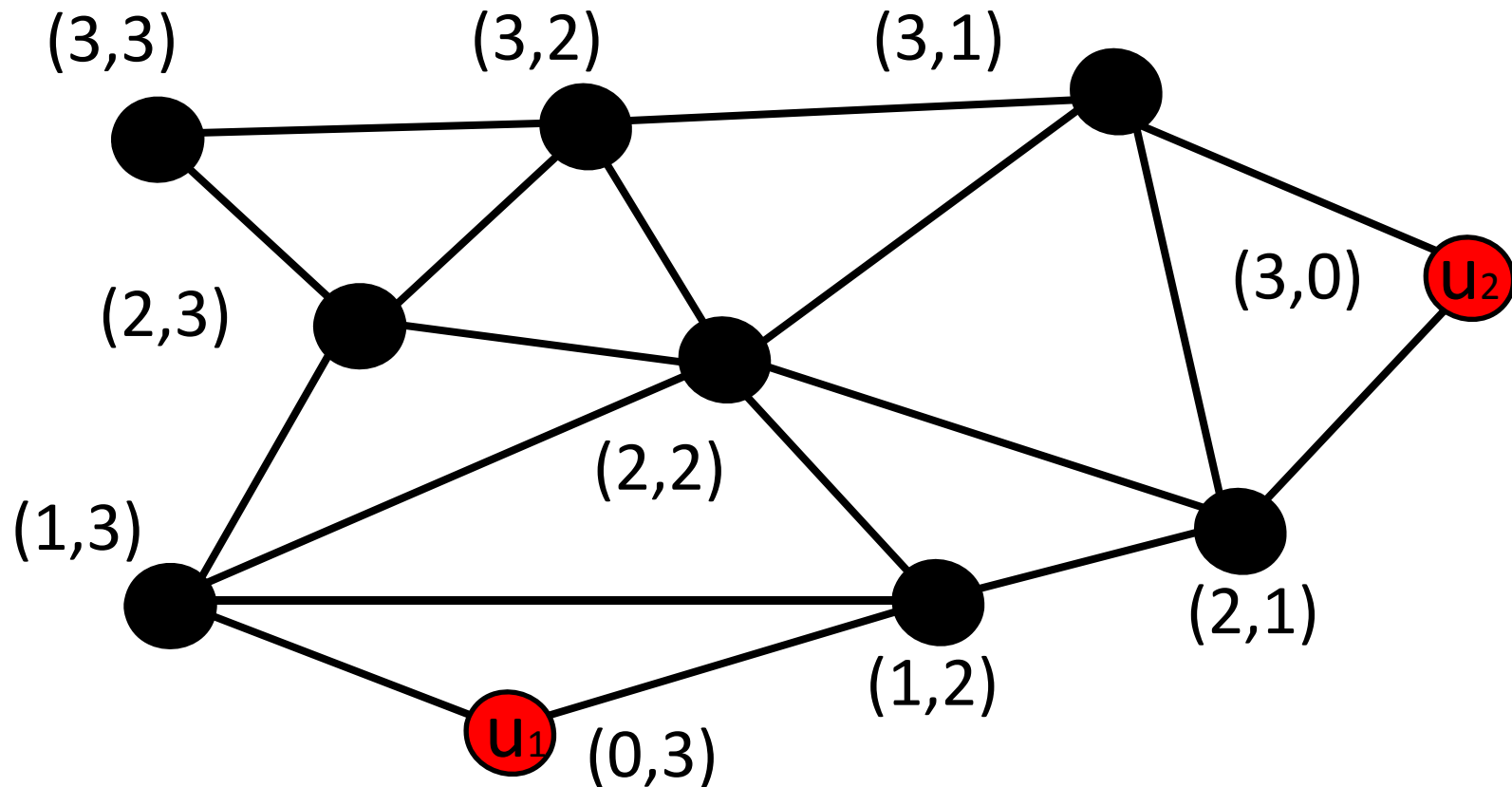
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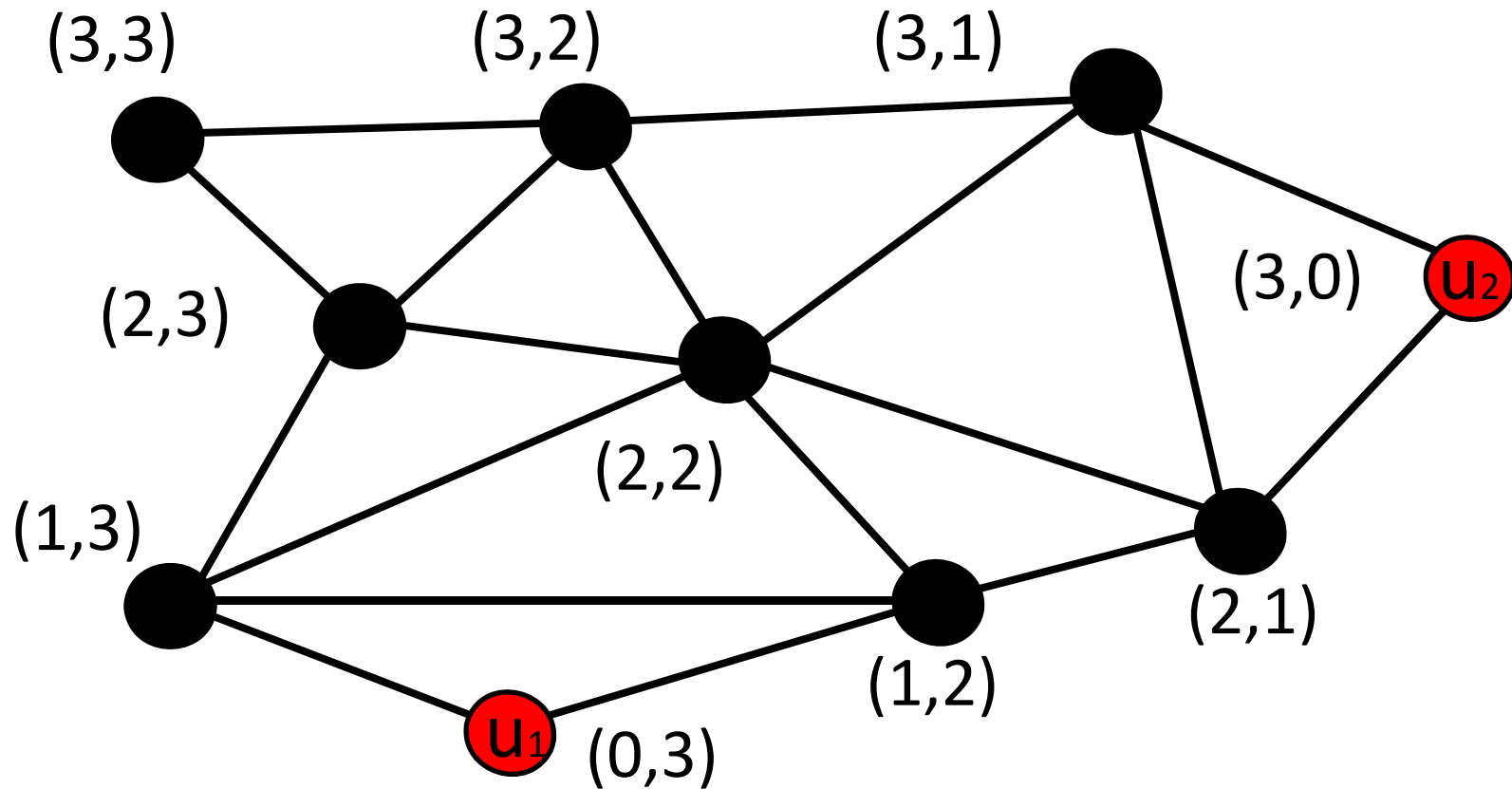
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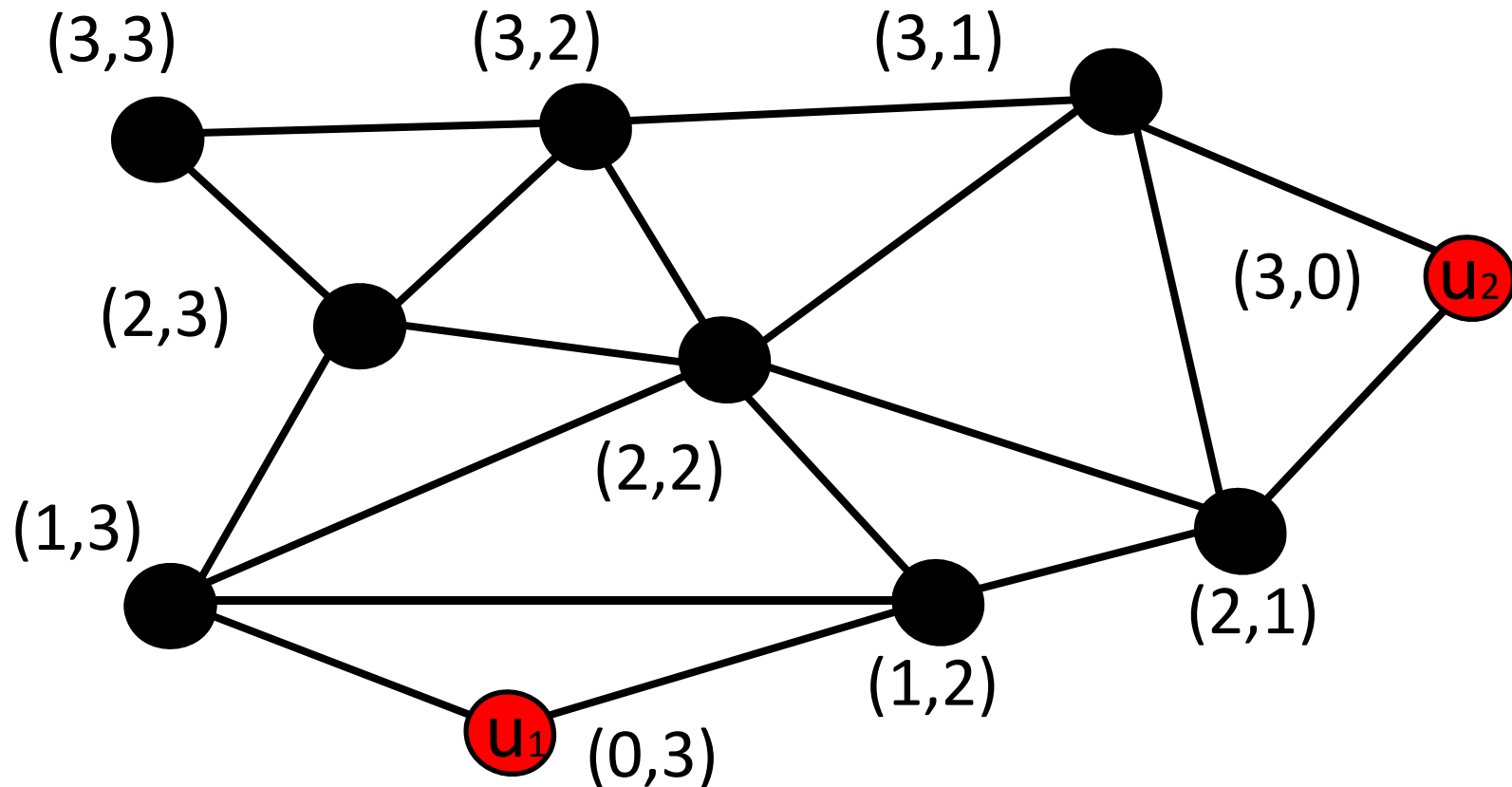
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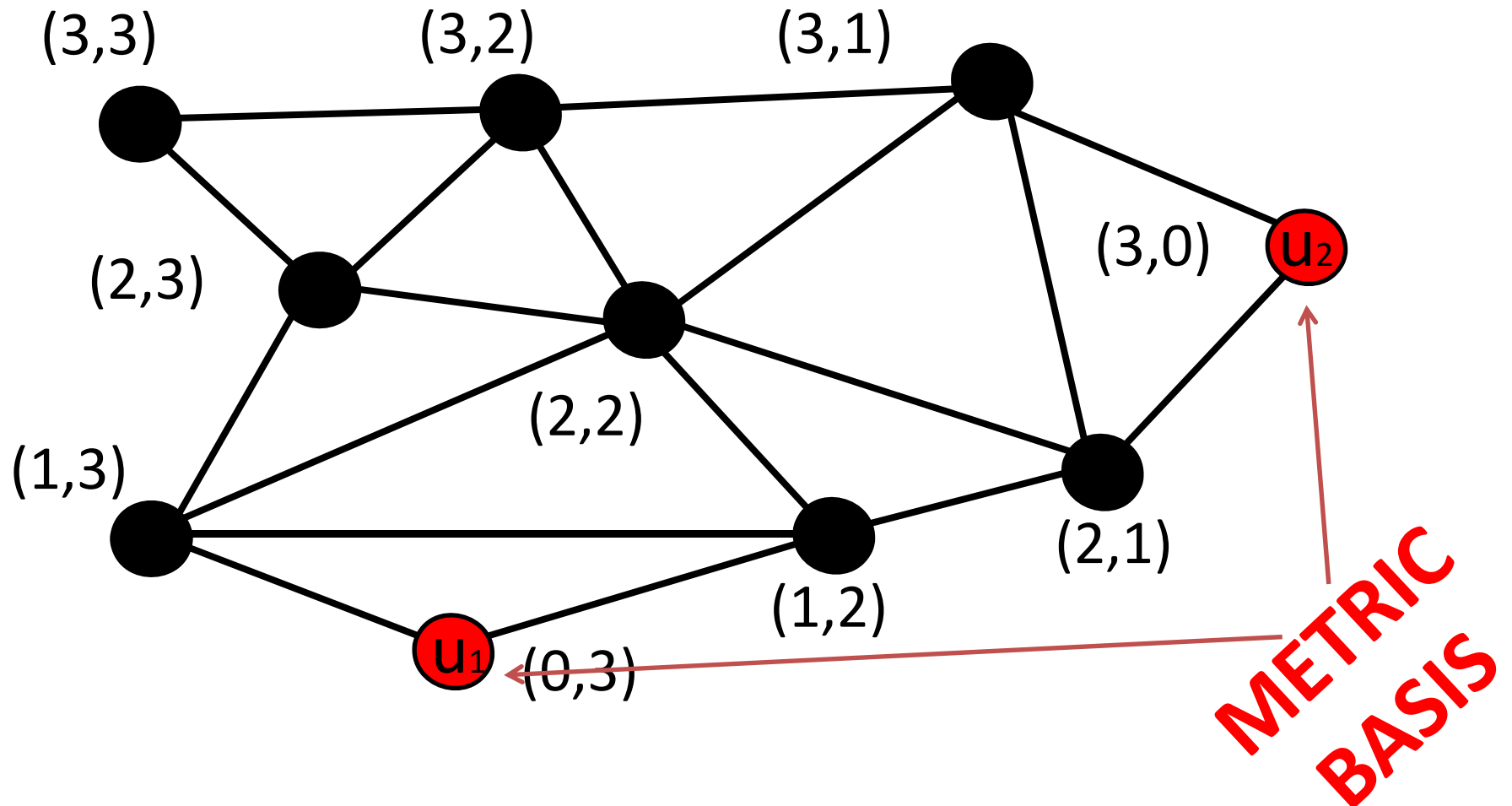
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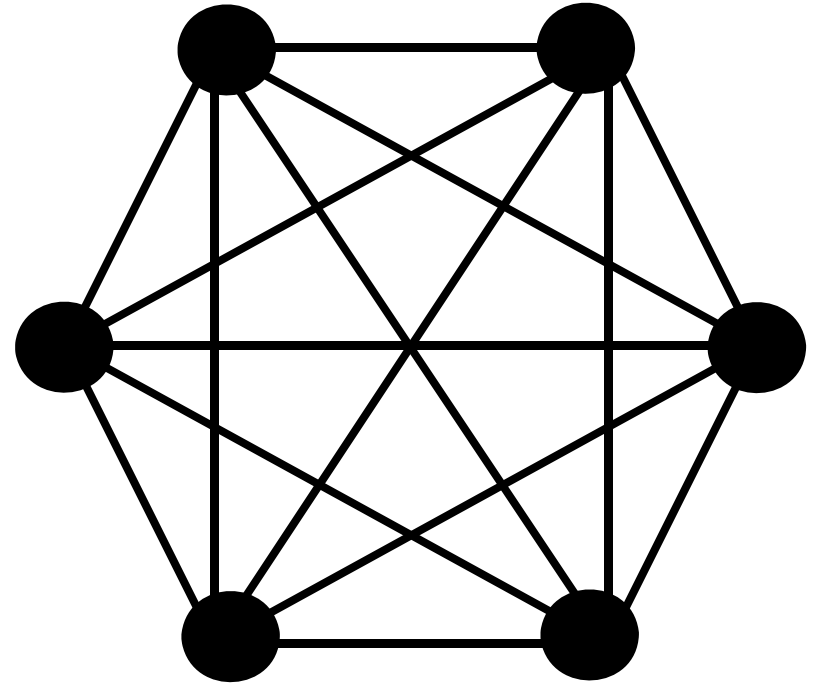
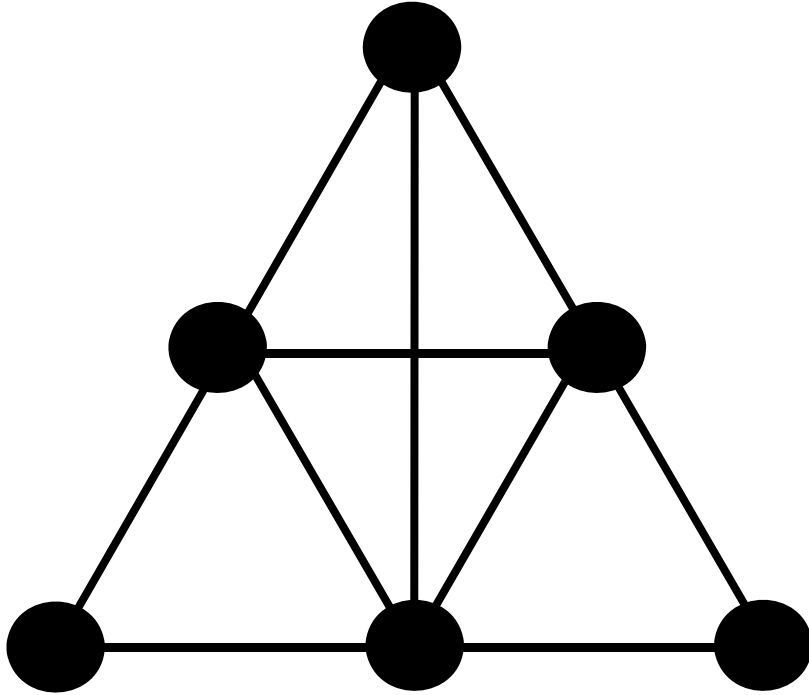
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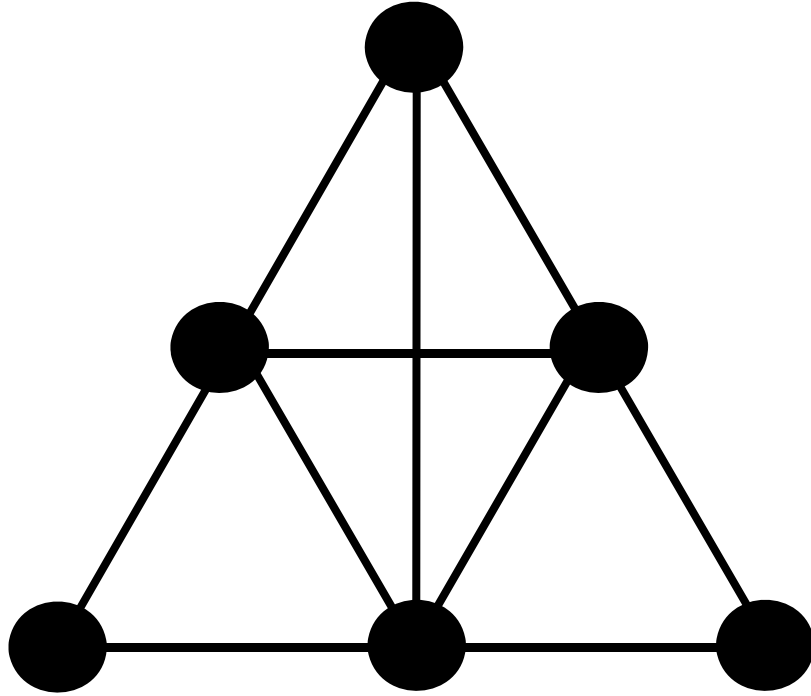


The number of resolving sets of a graph

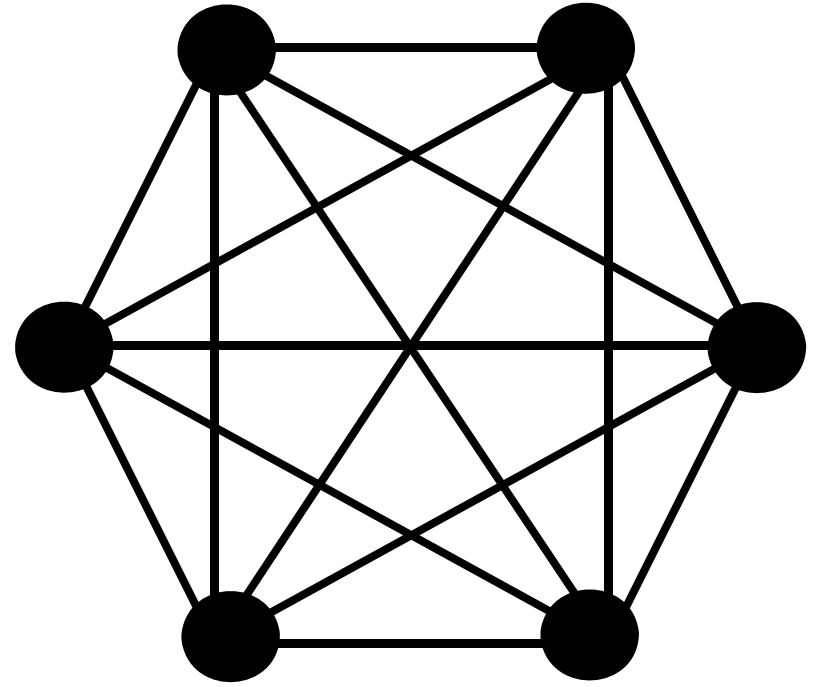
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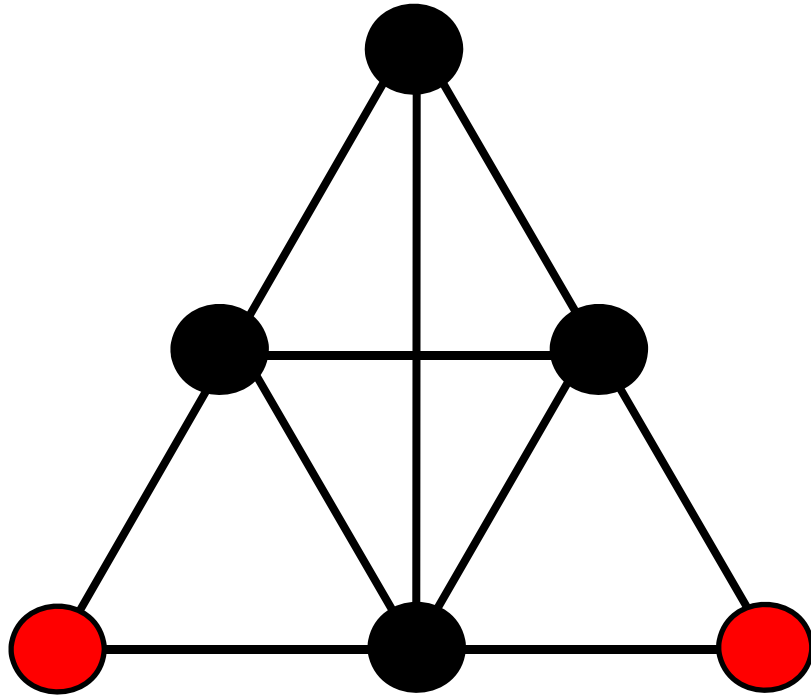


dim=2

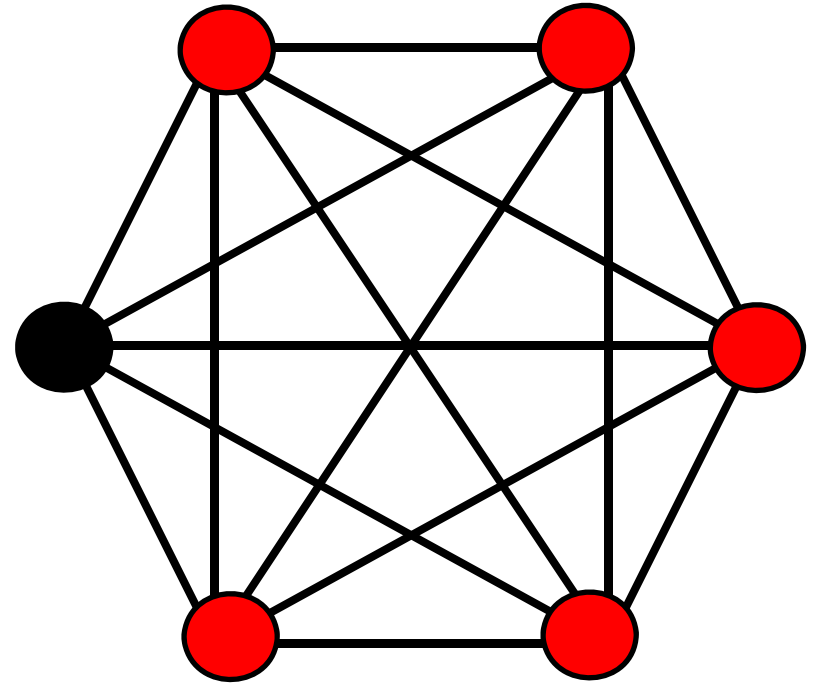


dim=5

# The number of resolving sets of a graph

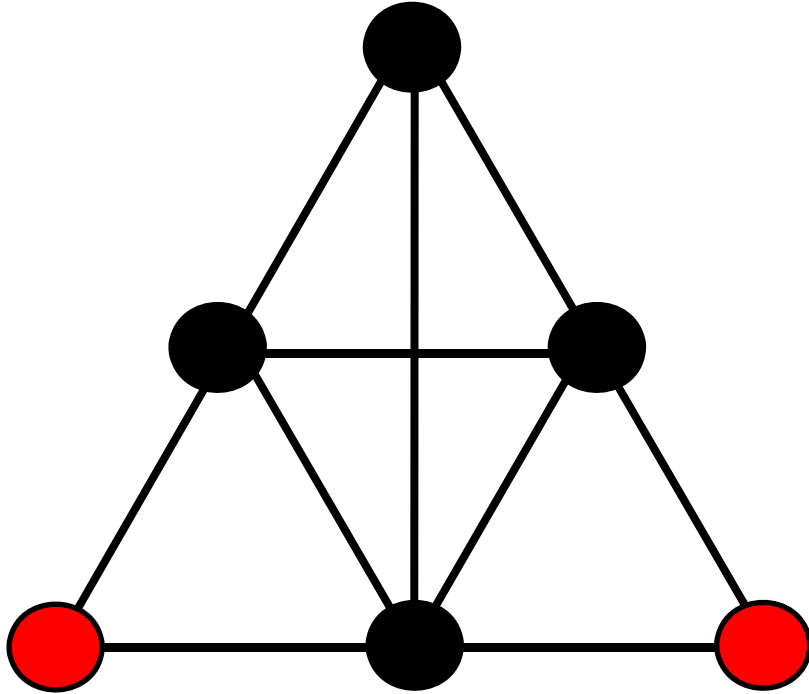


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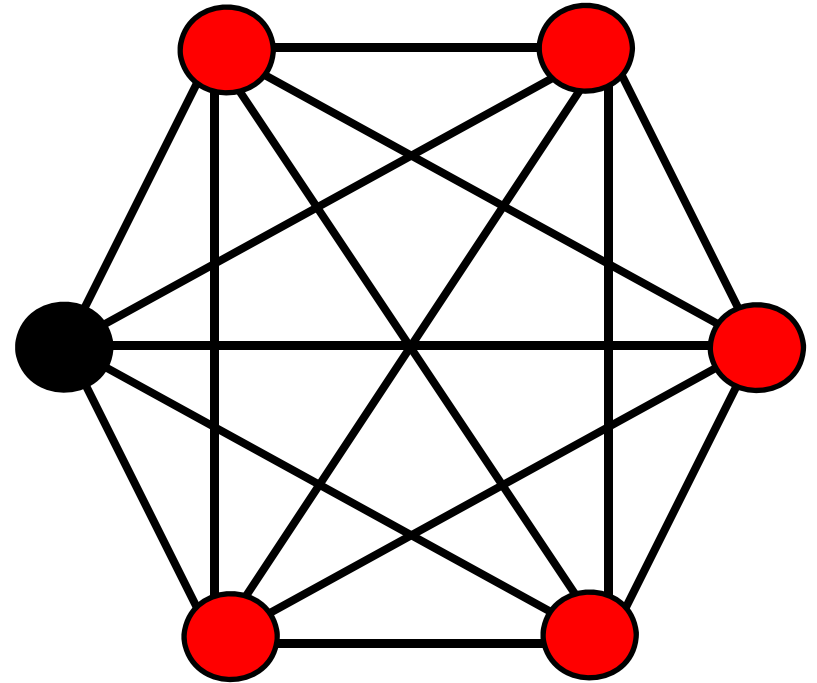
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# The number of resolving sets of a graph



$\dim=2$

$\# \text{ bases}=1$



$\dim=5$

$\# \text{ bases}=6$

# Graphs with “many” metric bases



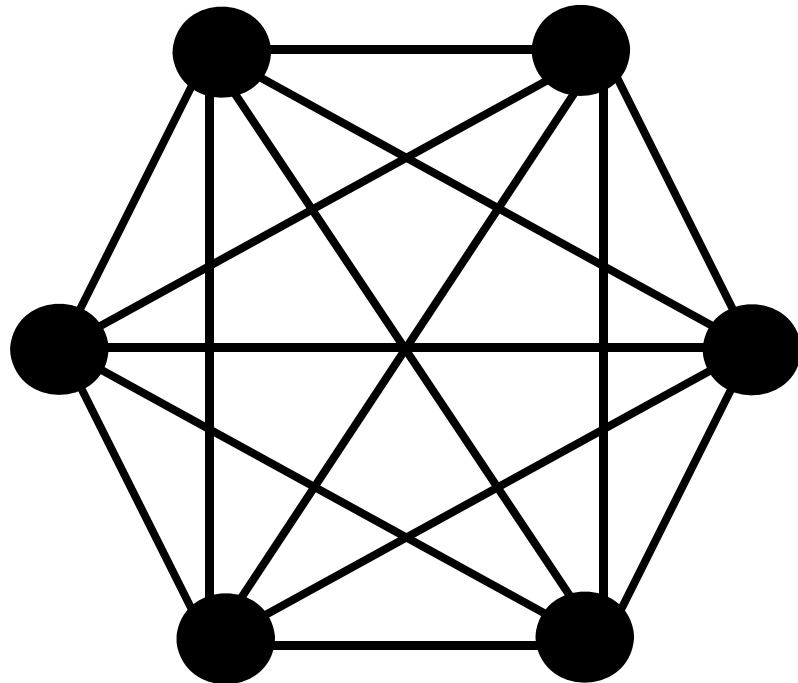
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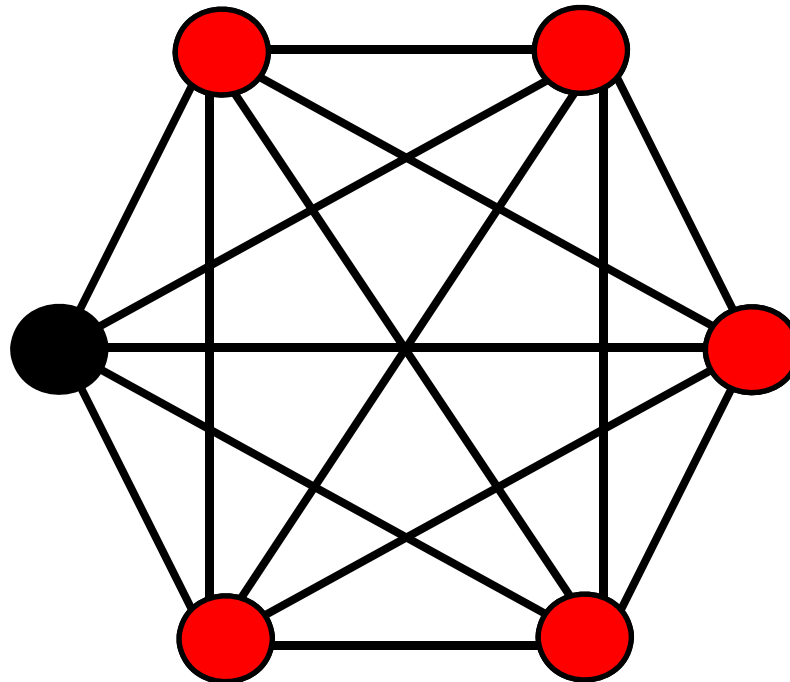
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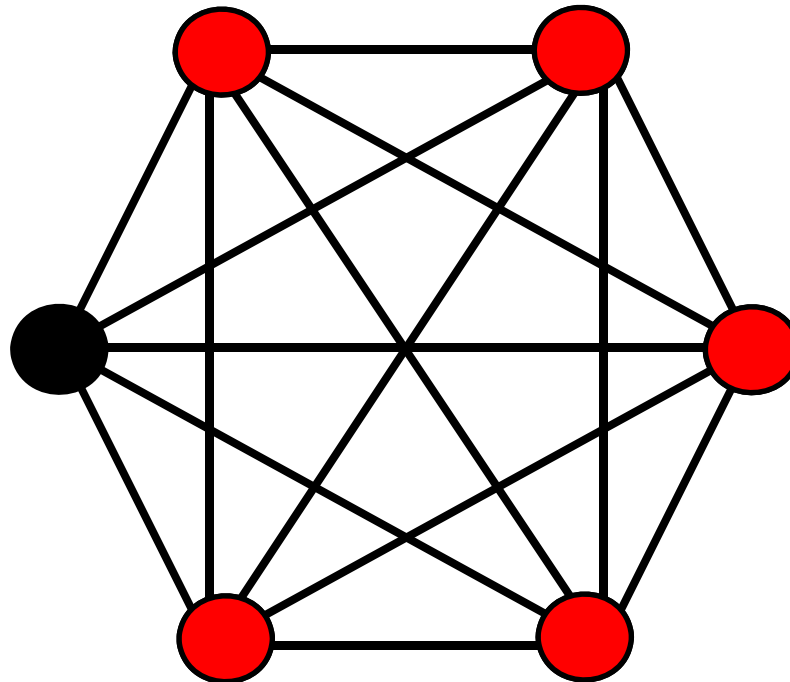
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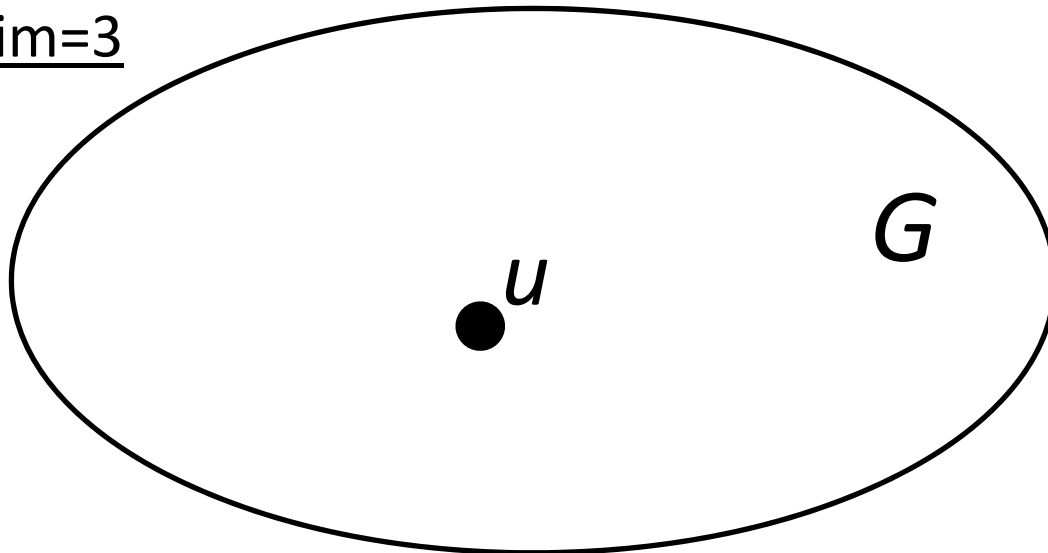
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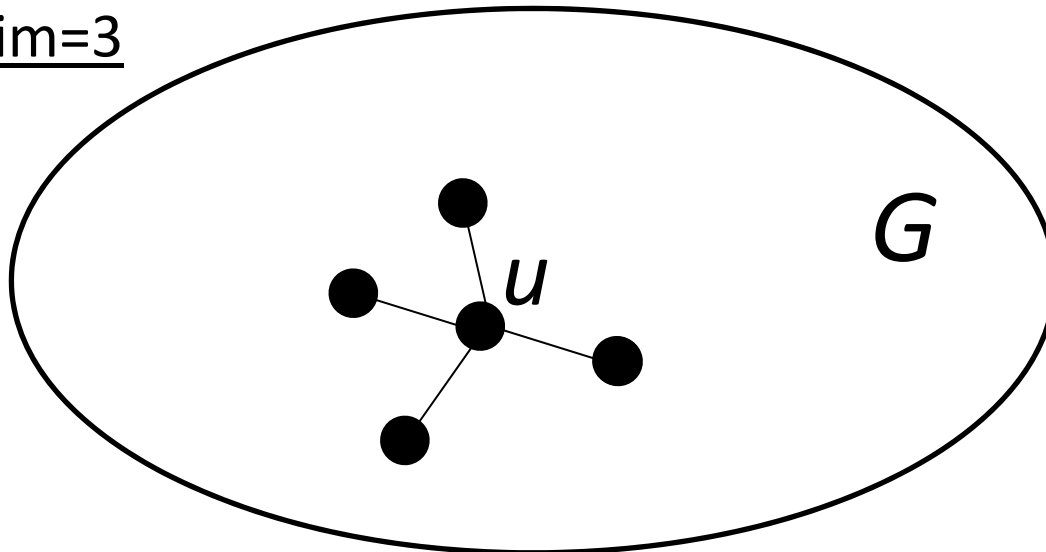
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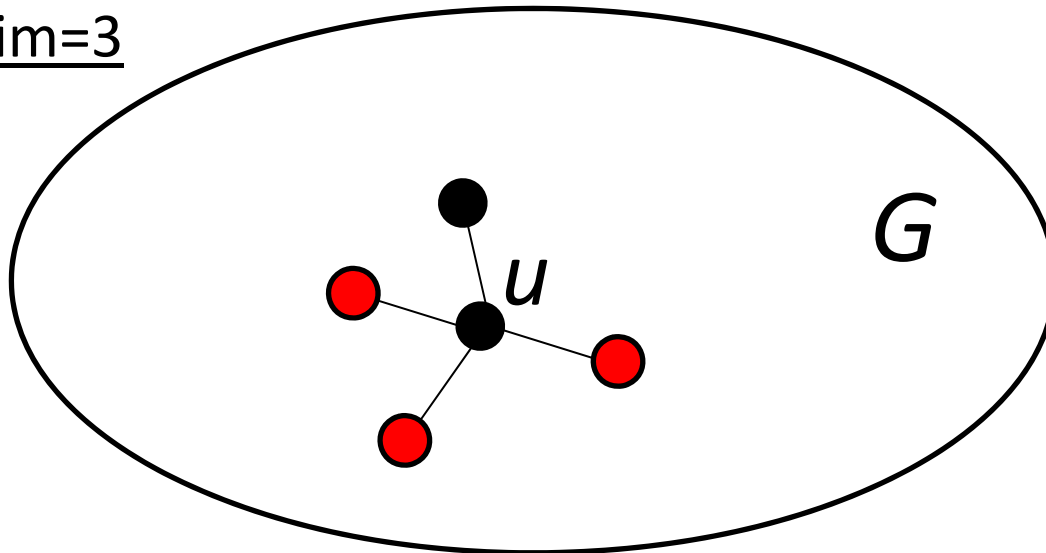
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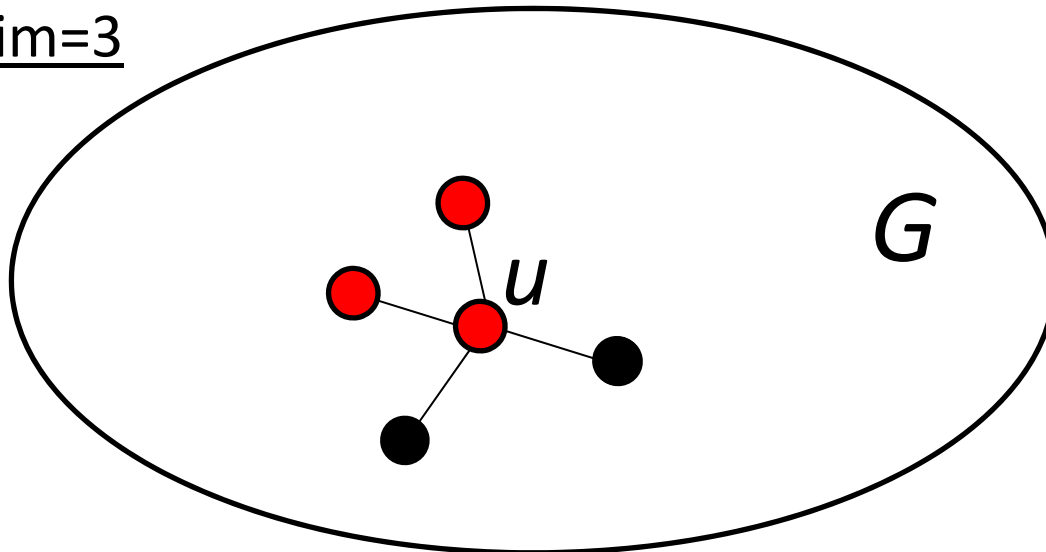
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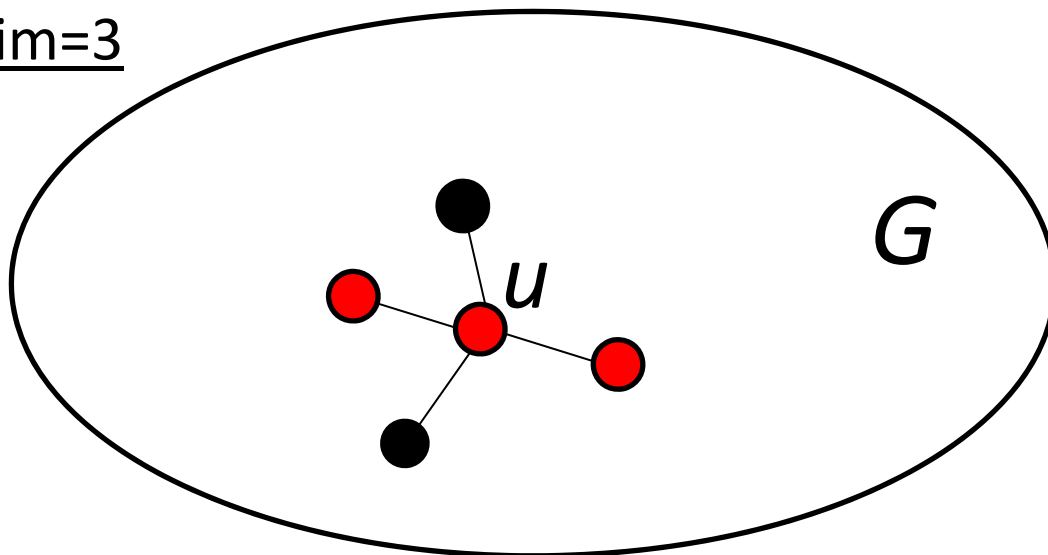
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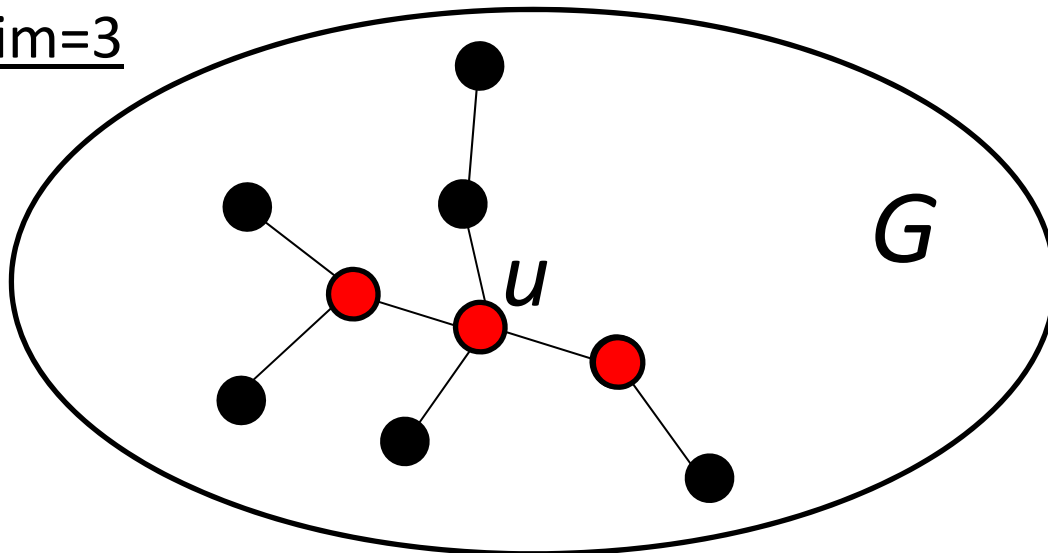
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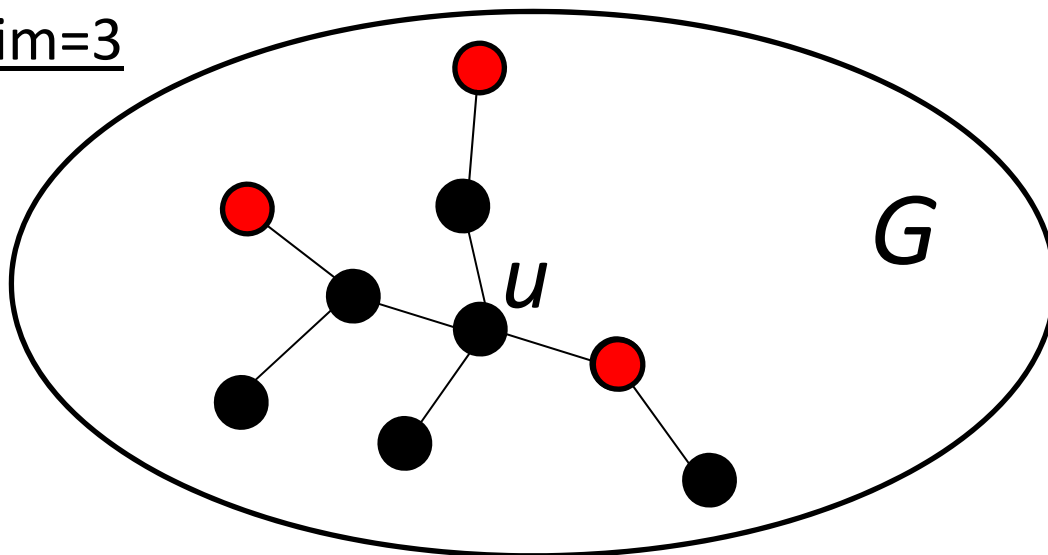
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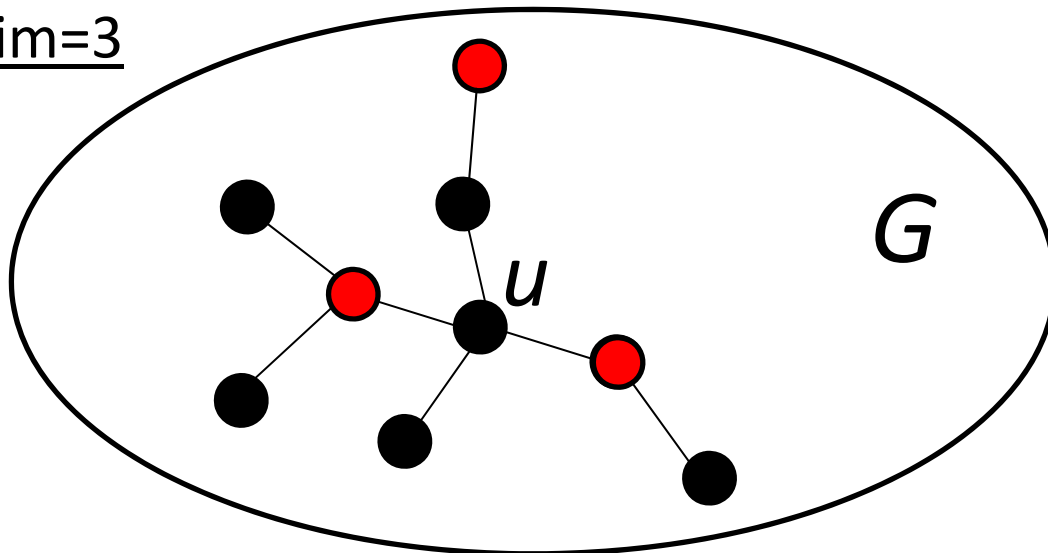
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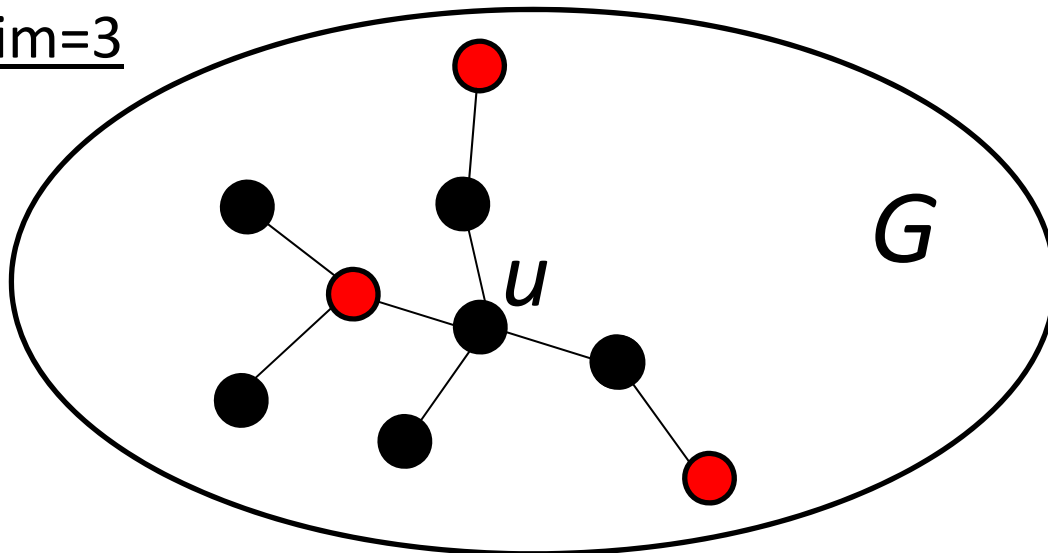
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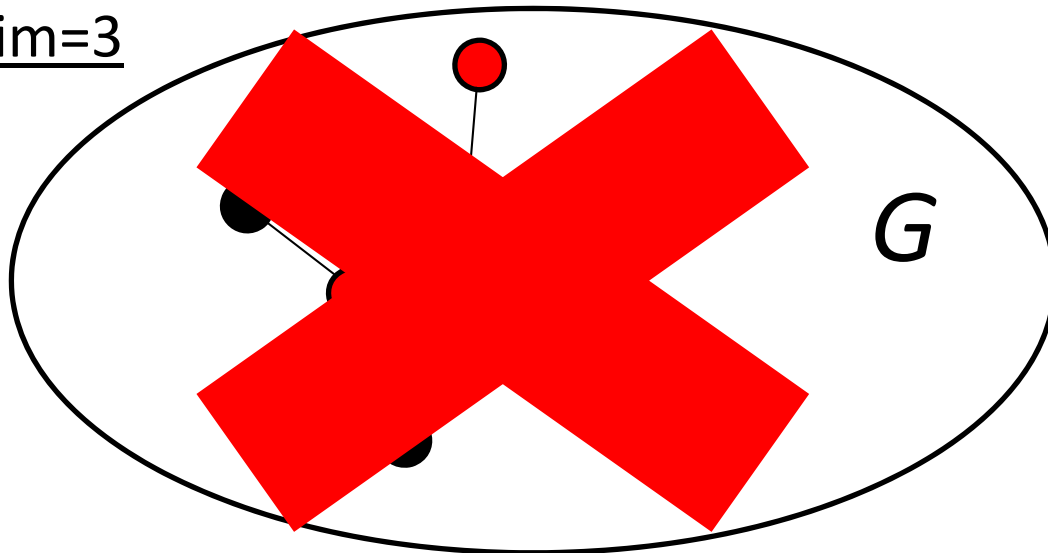
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$\mathcal{P}_2(V)$



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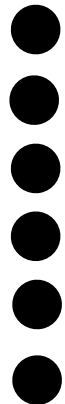
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$\mathcal{P}_2(V)$



NON-BASES

$\mathcal{P}_2(V)$



NON-DISTINGUISHED  
PAIRS

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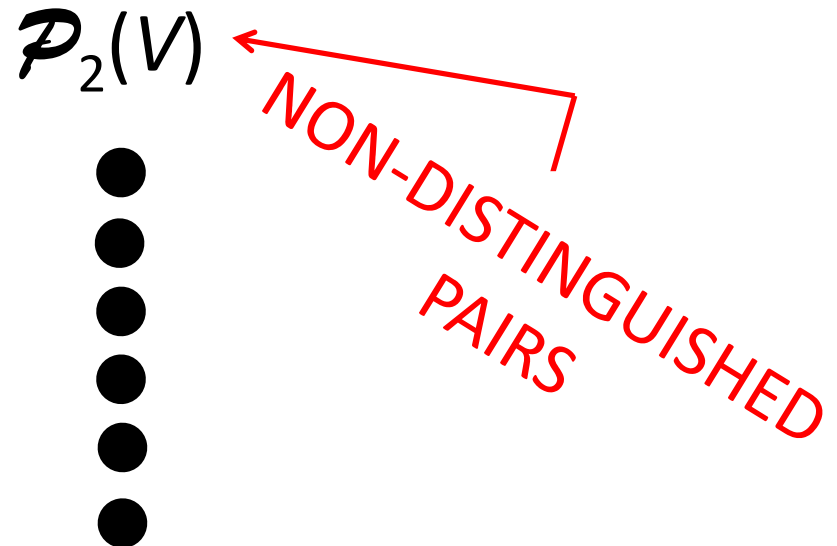
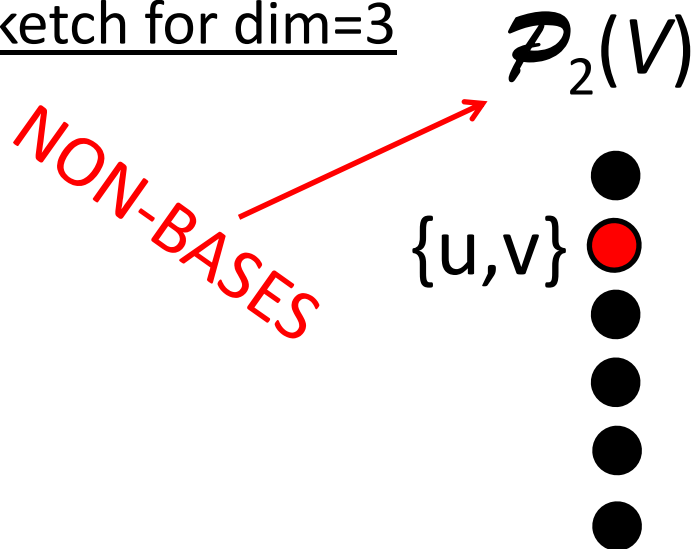
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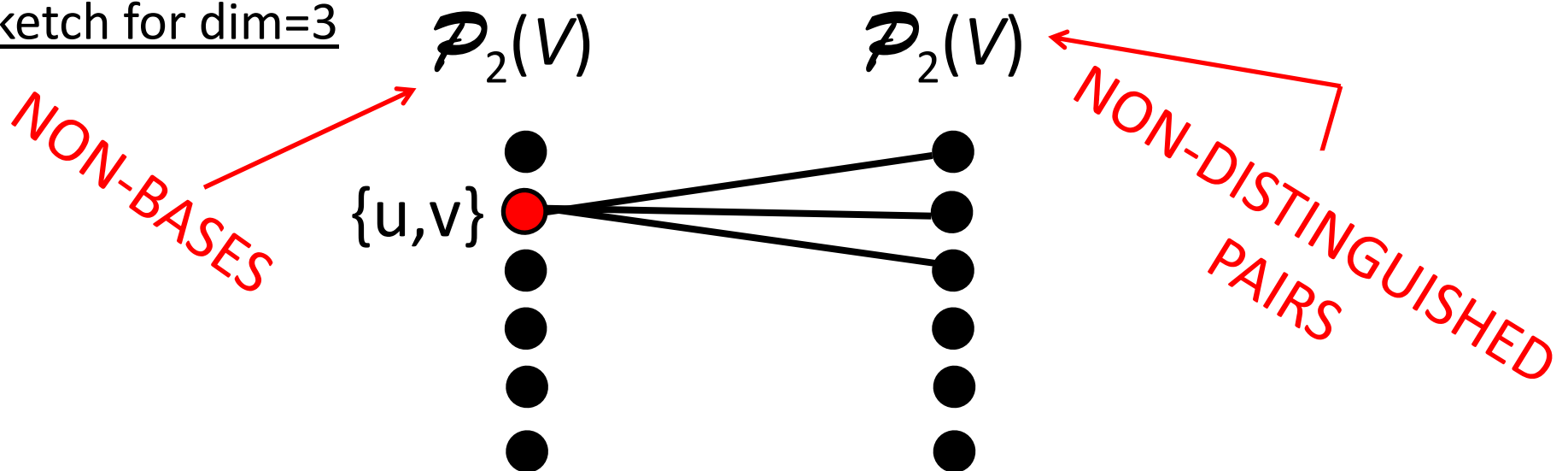
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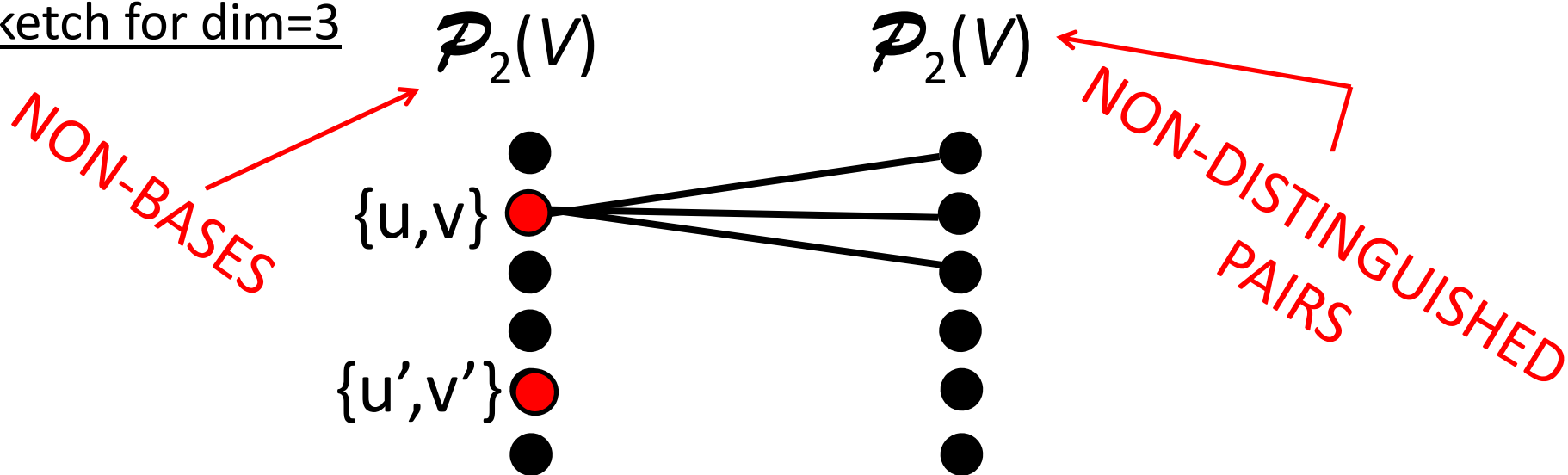
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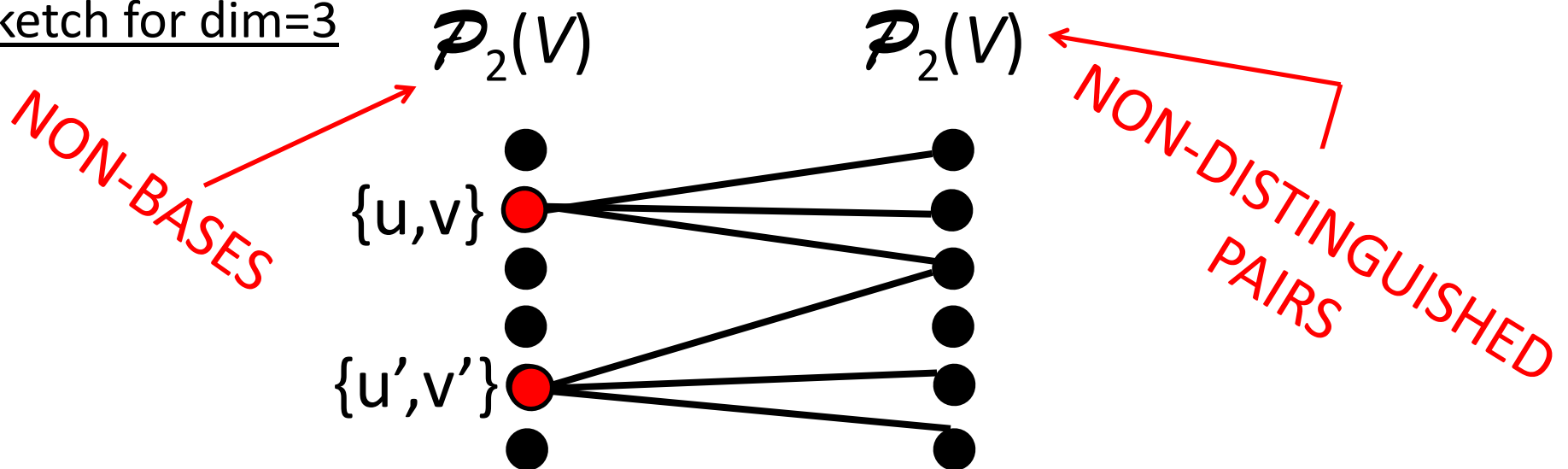
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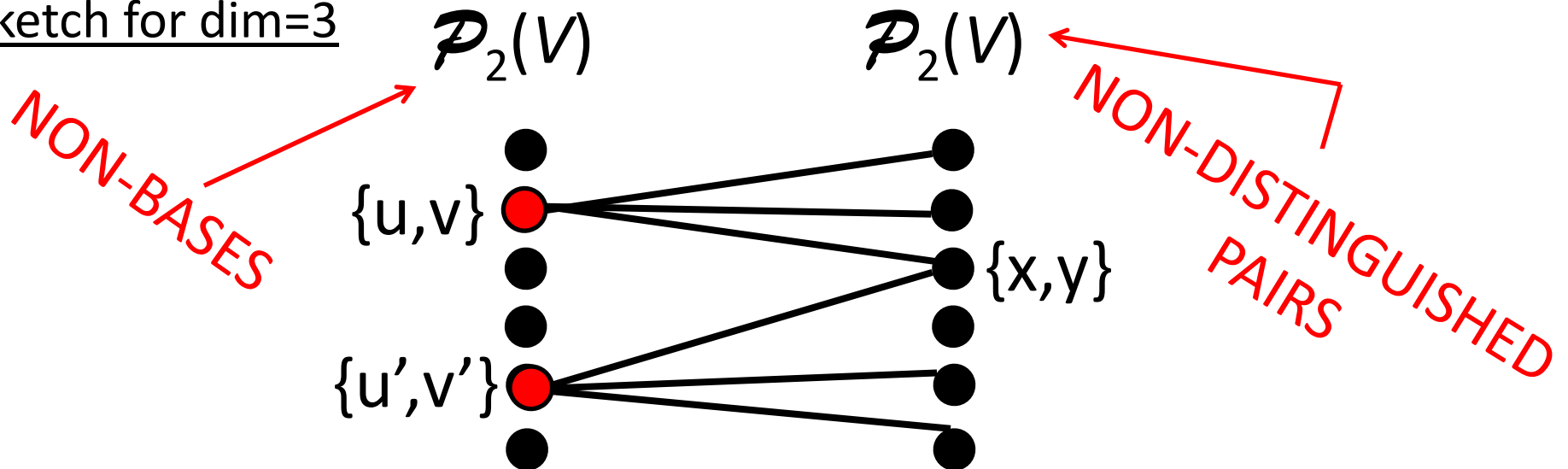
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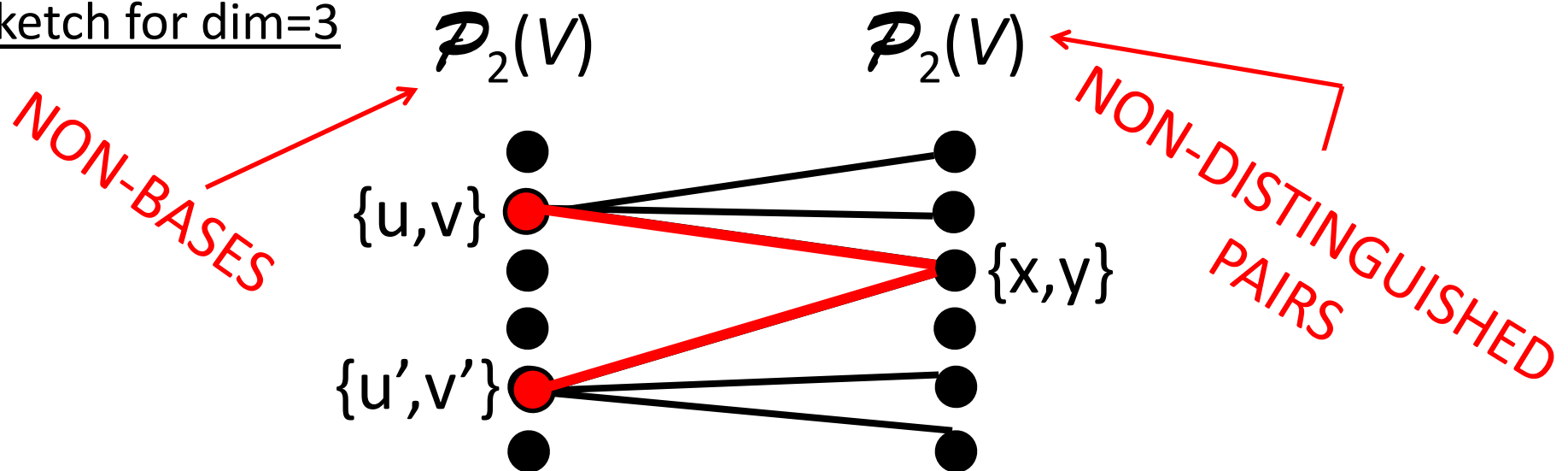
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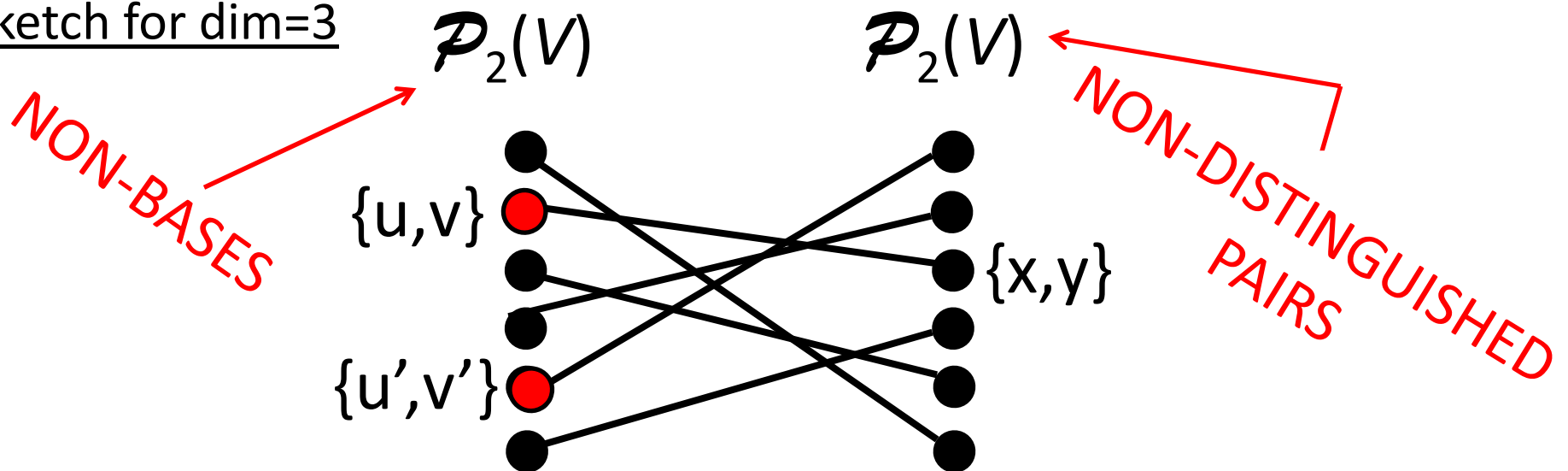
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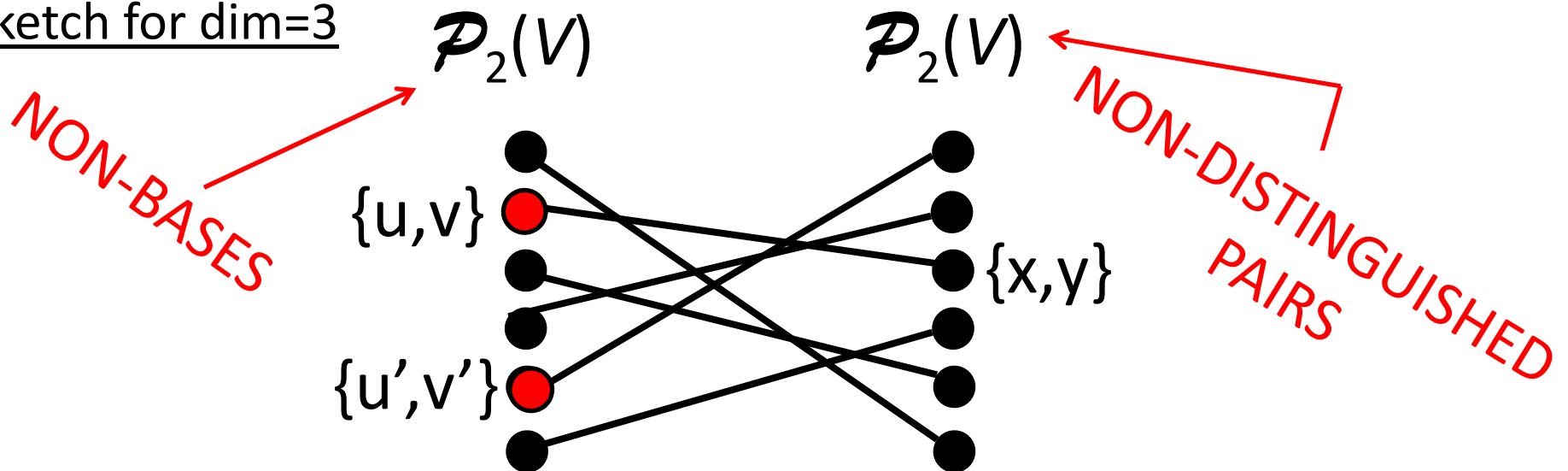
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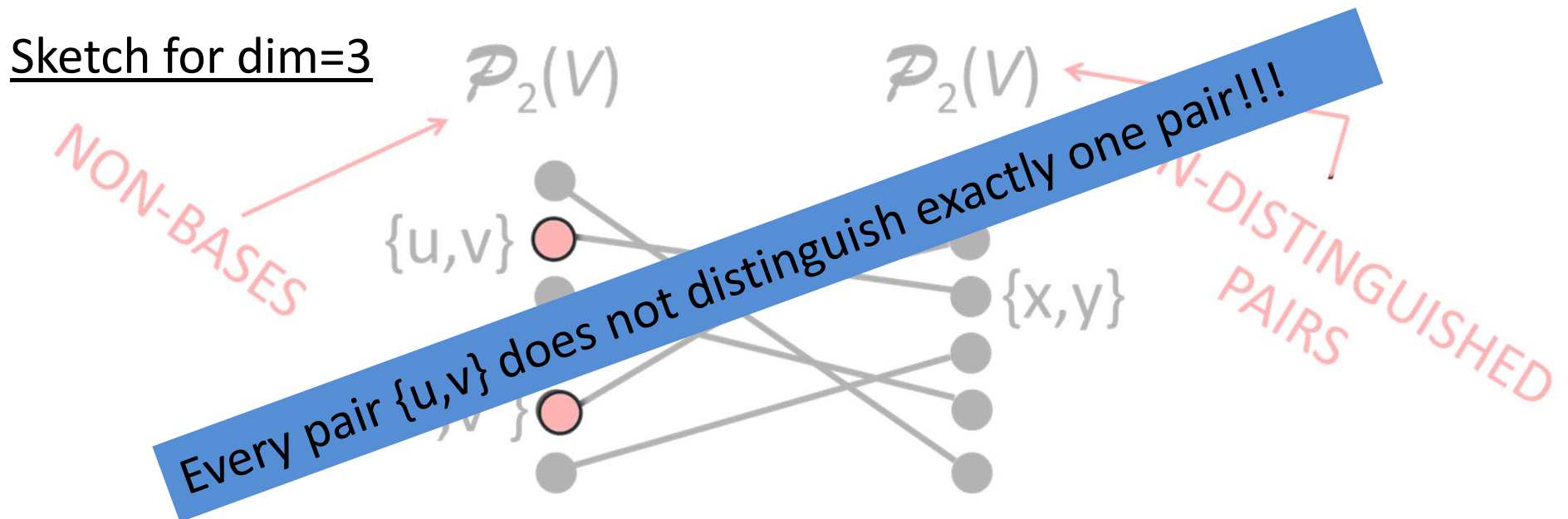
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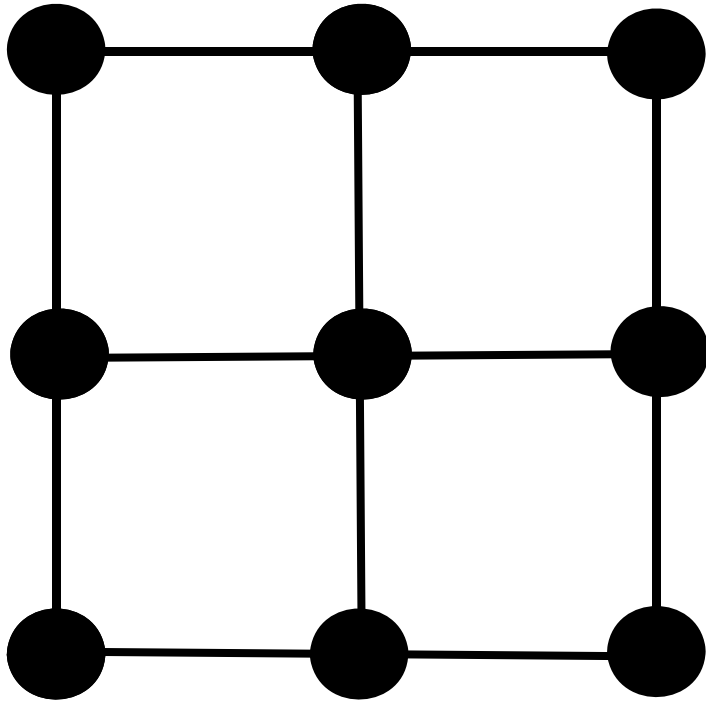
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$\dim^+(G)$  = maximum size of a minimal resolving set



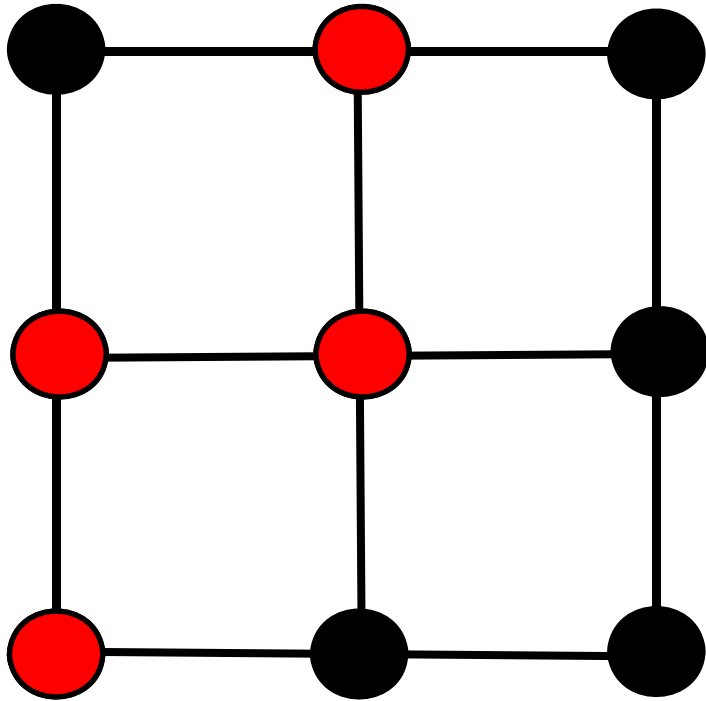
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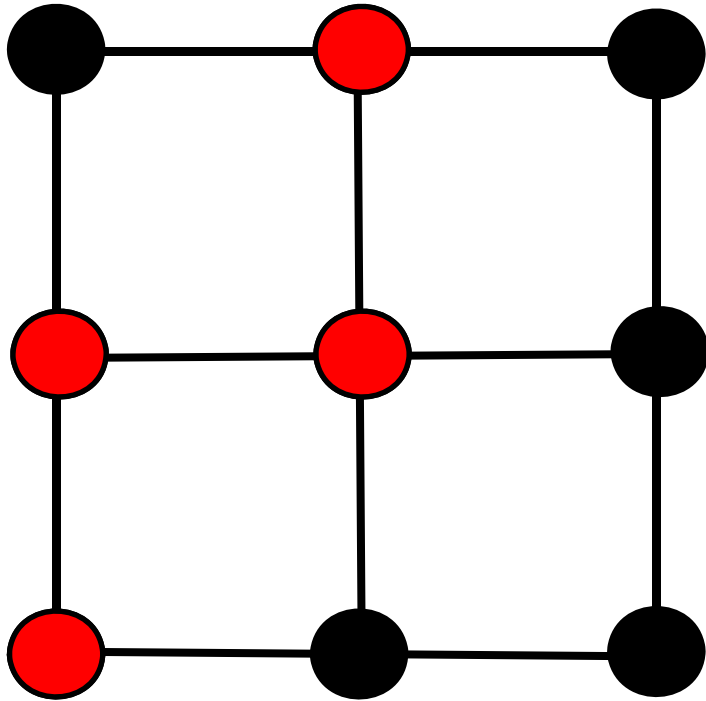
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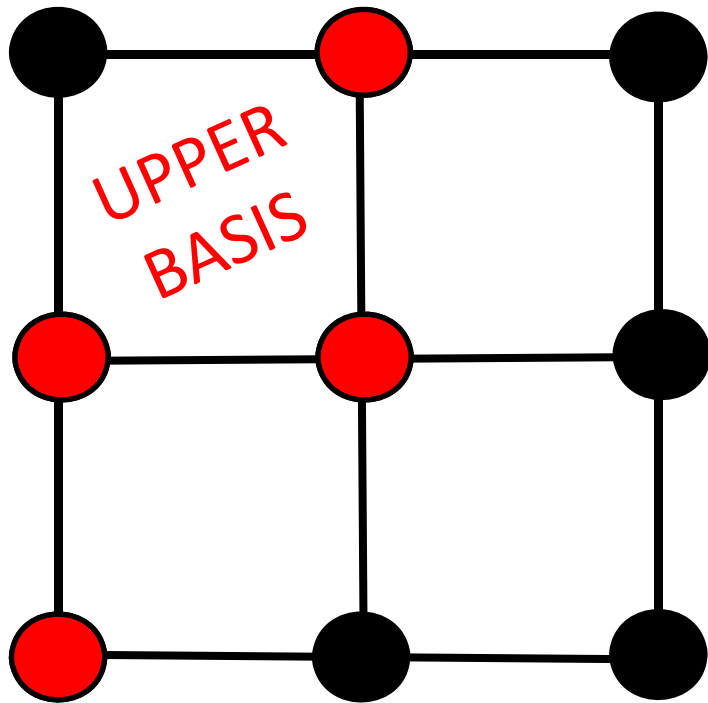
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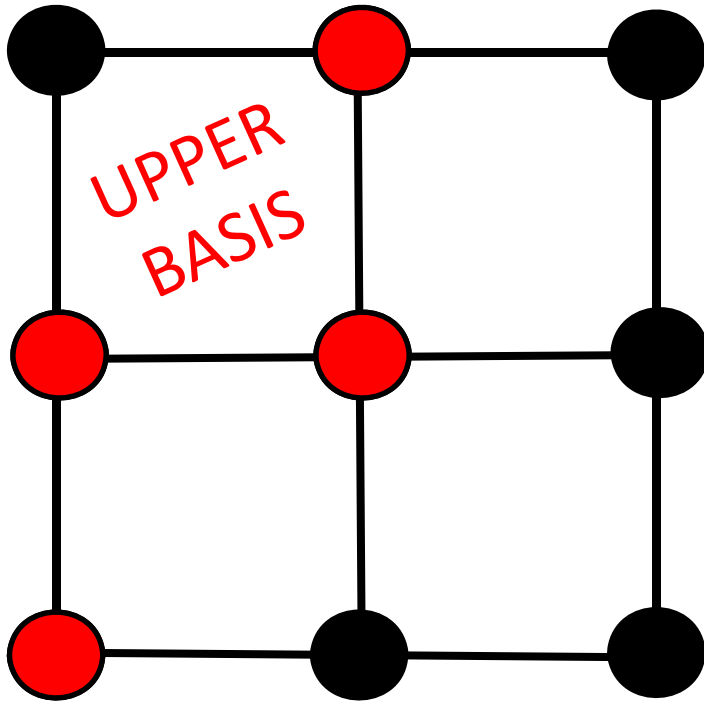


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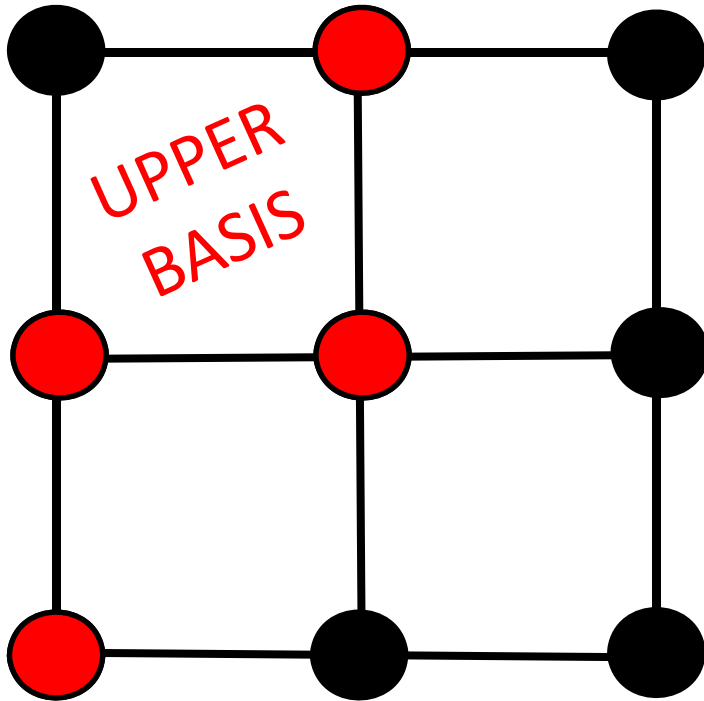


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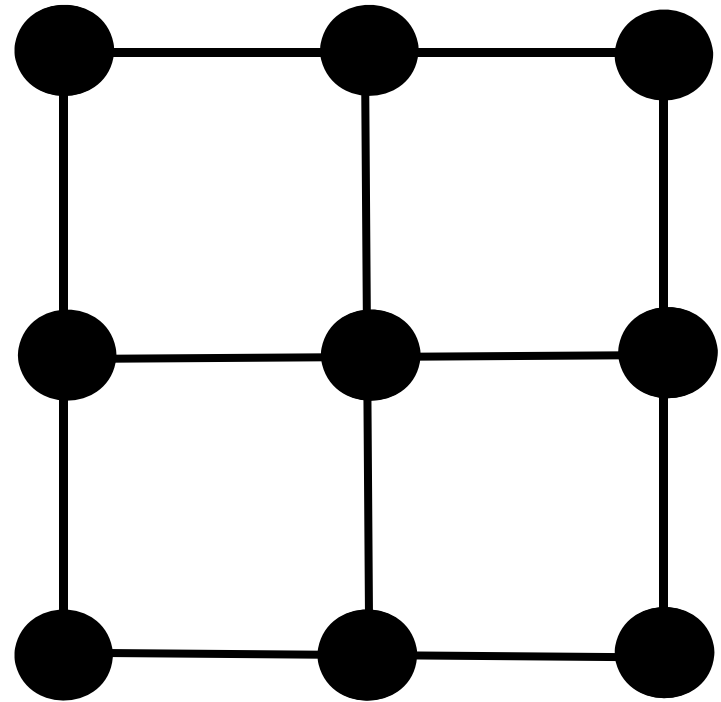
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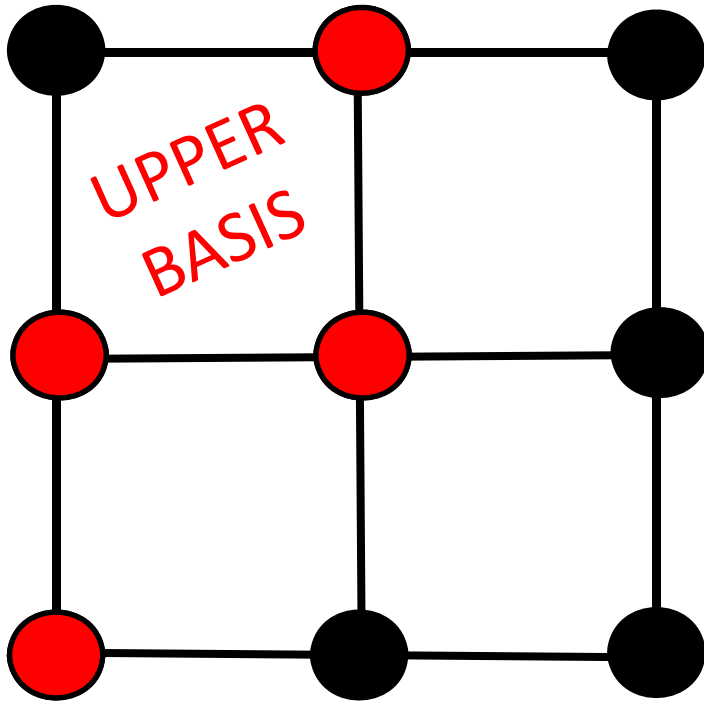
$\dim^+(G) = 4$



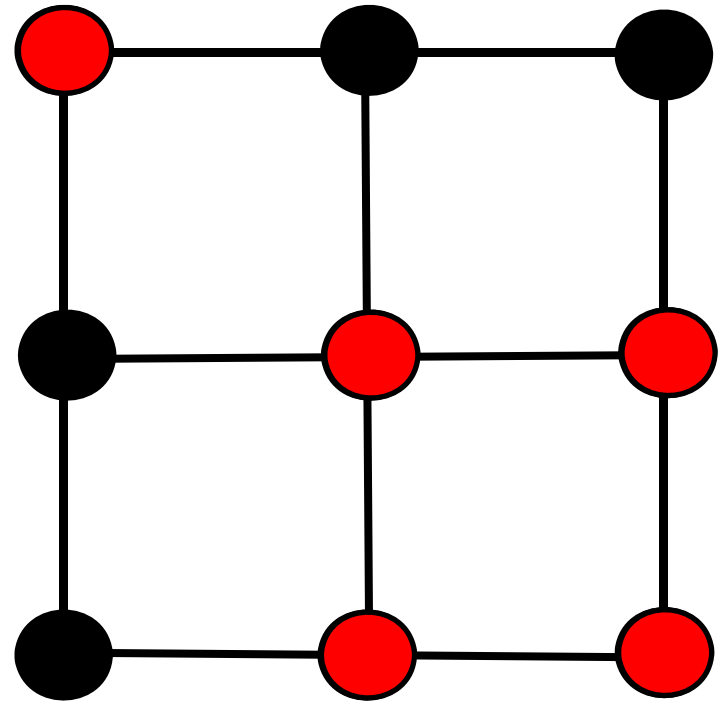
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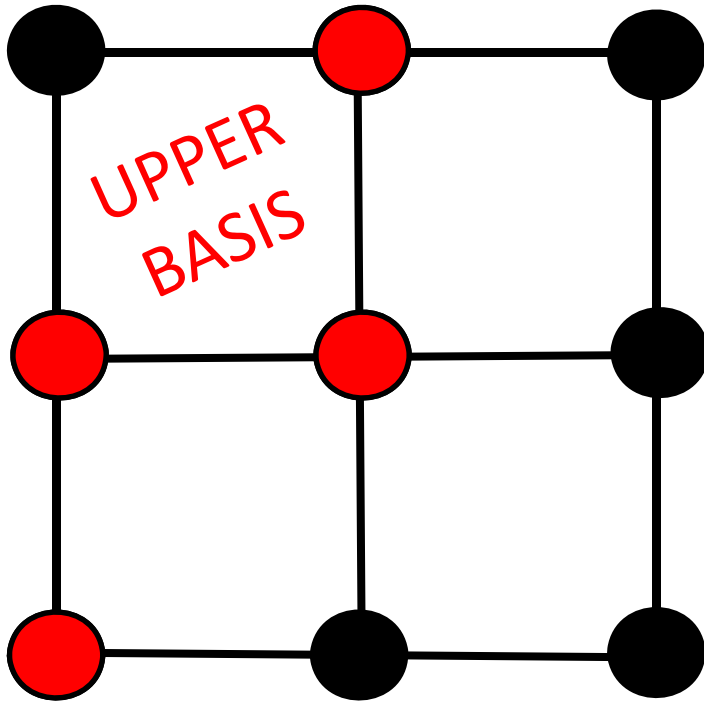
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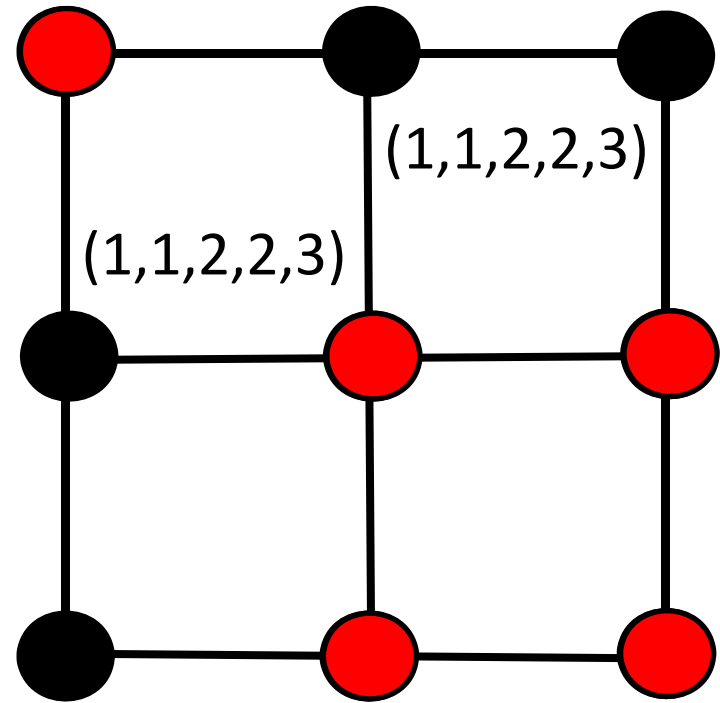
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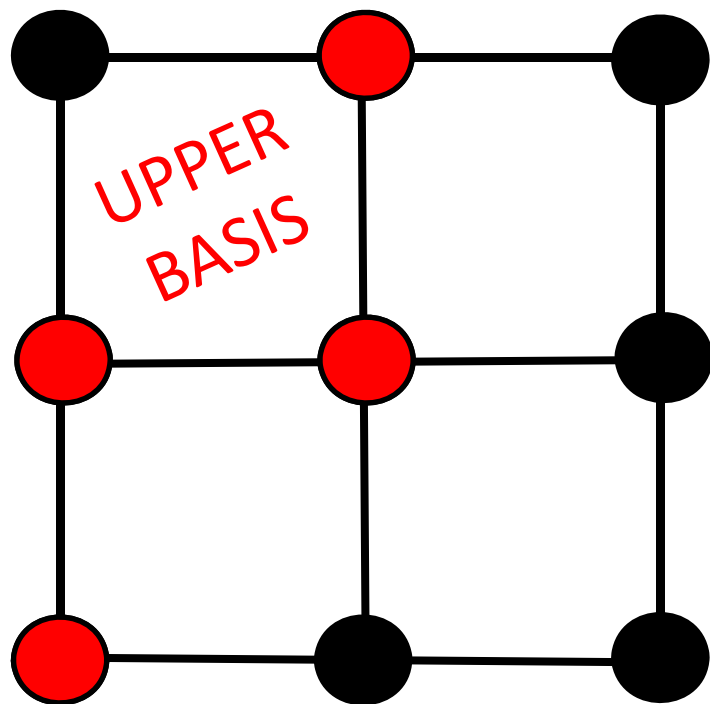
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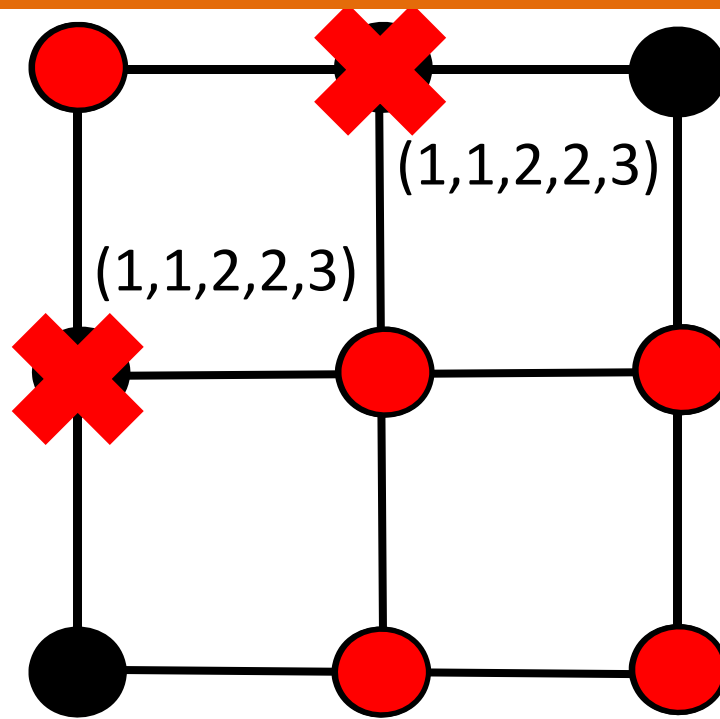
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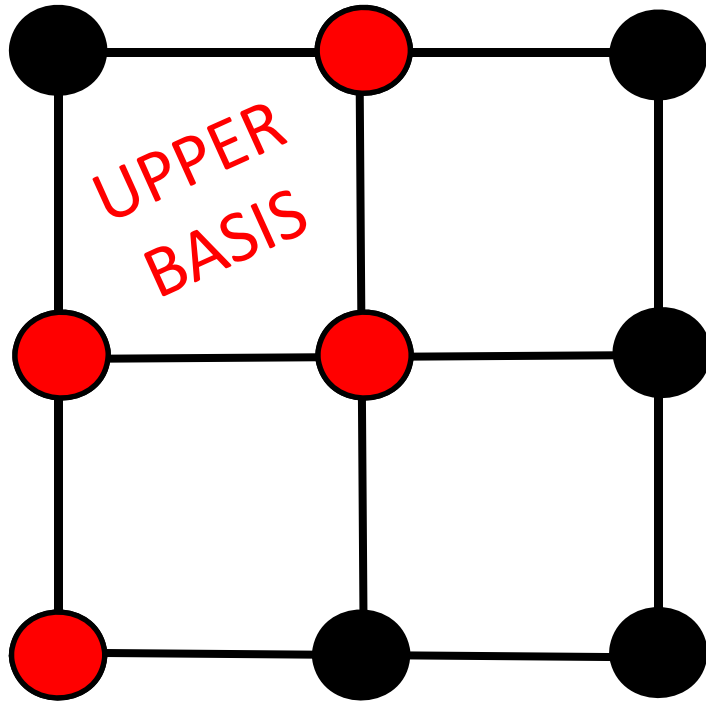
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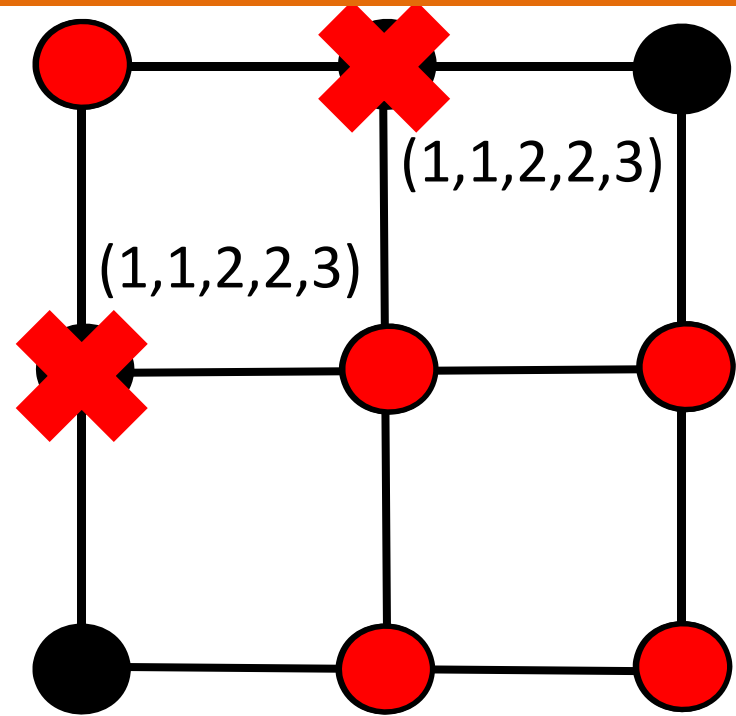
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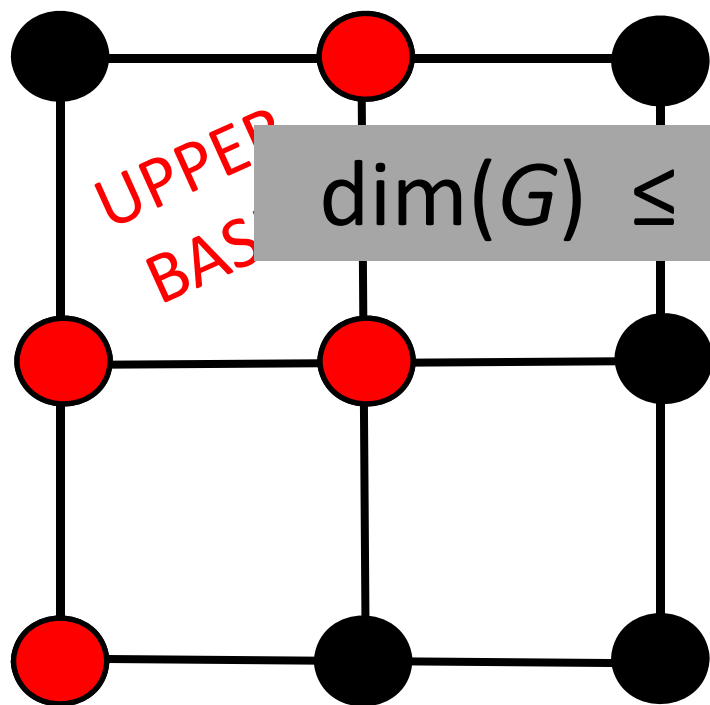


$$\text{res}(G) = 6$$

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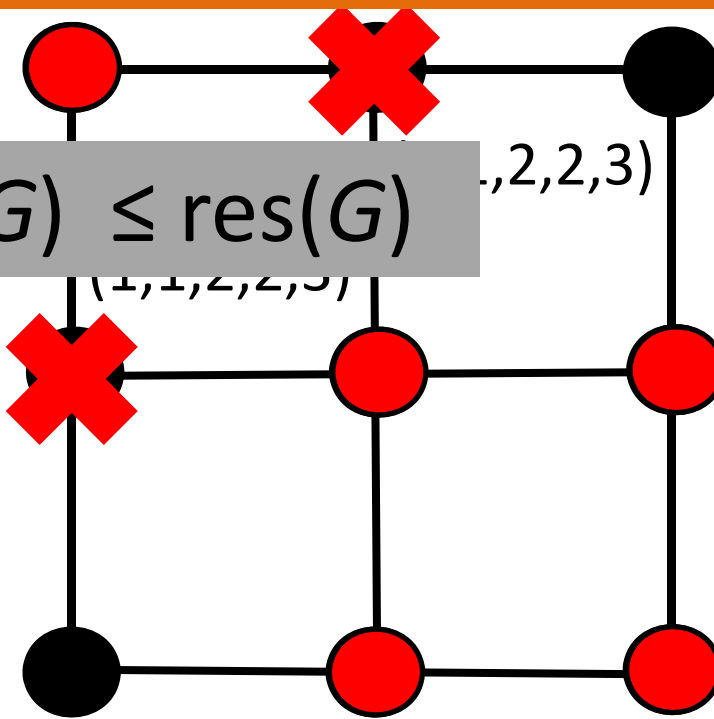
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$$\dim^+(G) = 4$$

$$\dim(G) \leq \dim^+(G) \leq \text{res}(G)$$



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(1, 2, 2, 3)

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# Realizability

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||  
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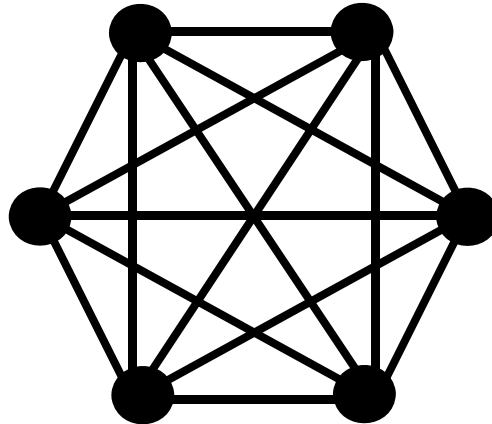


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$\equiv$   
a



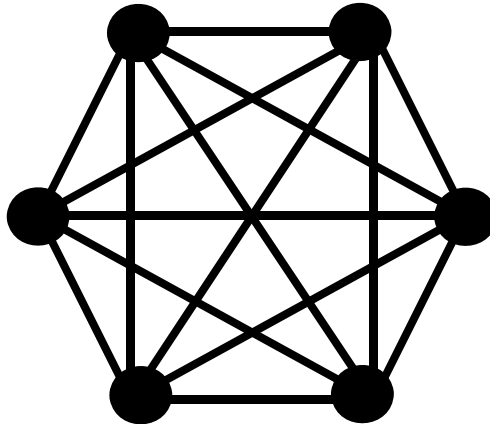
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[Chartrand et al., 2000]

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$\parallel$   
a

$$\dim(K_n) = n - 1$$



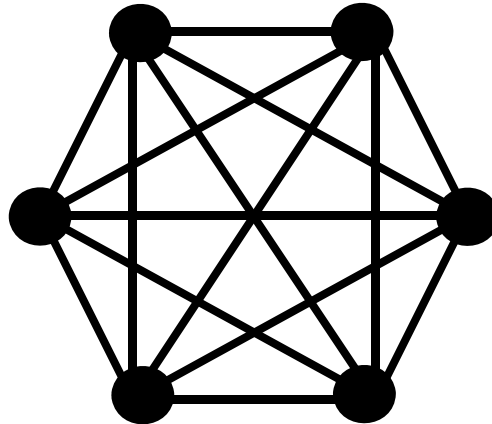
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$\equiv$   
C

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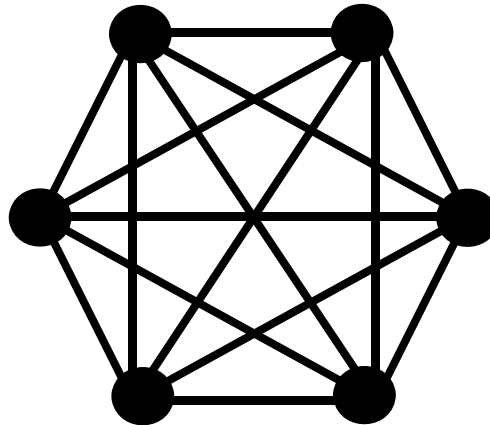
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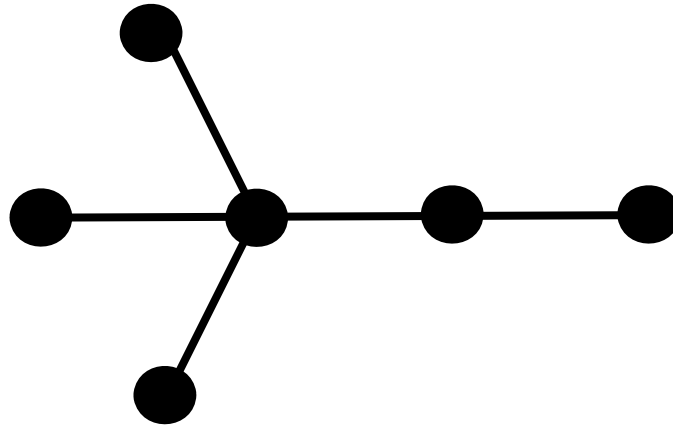
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a

$\parallel$   
c



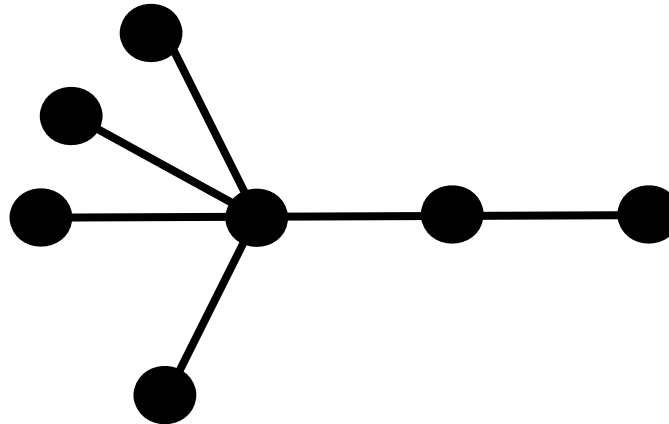
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$\parallel$   
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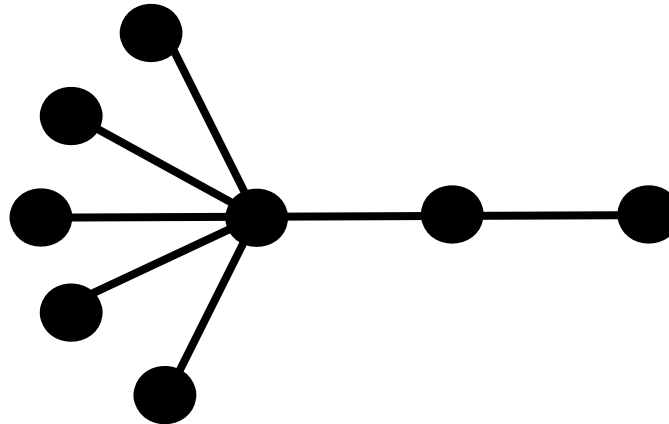
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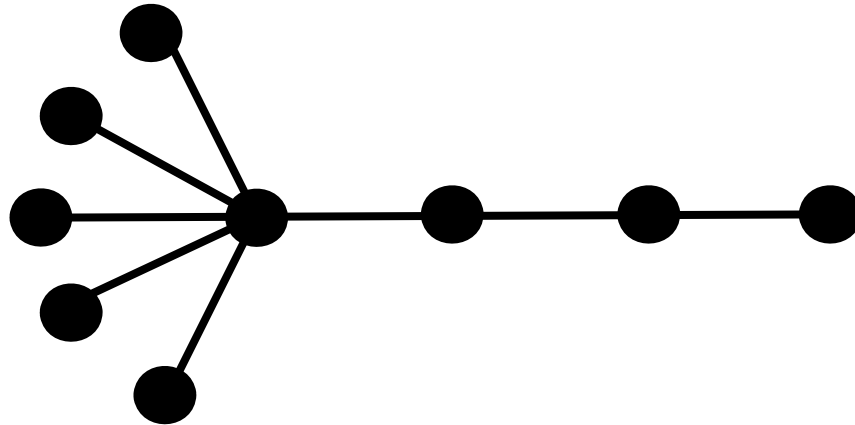
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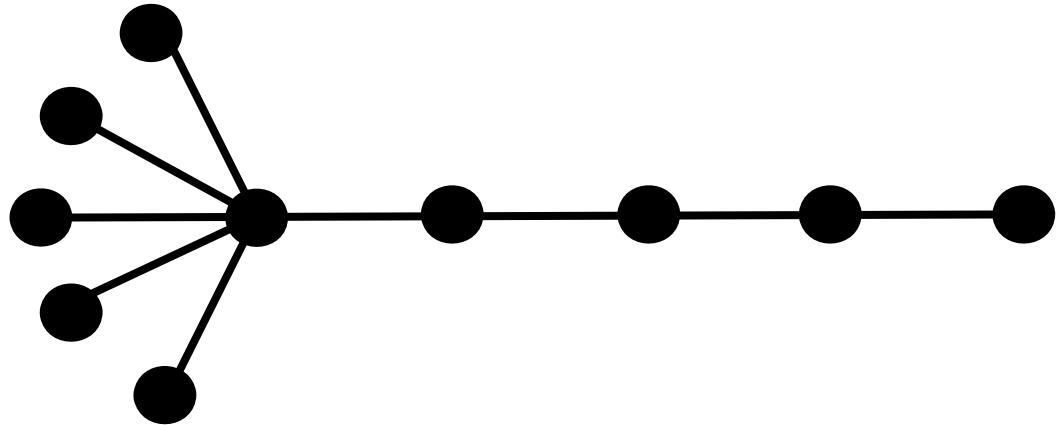
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$\parallel$   
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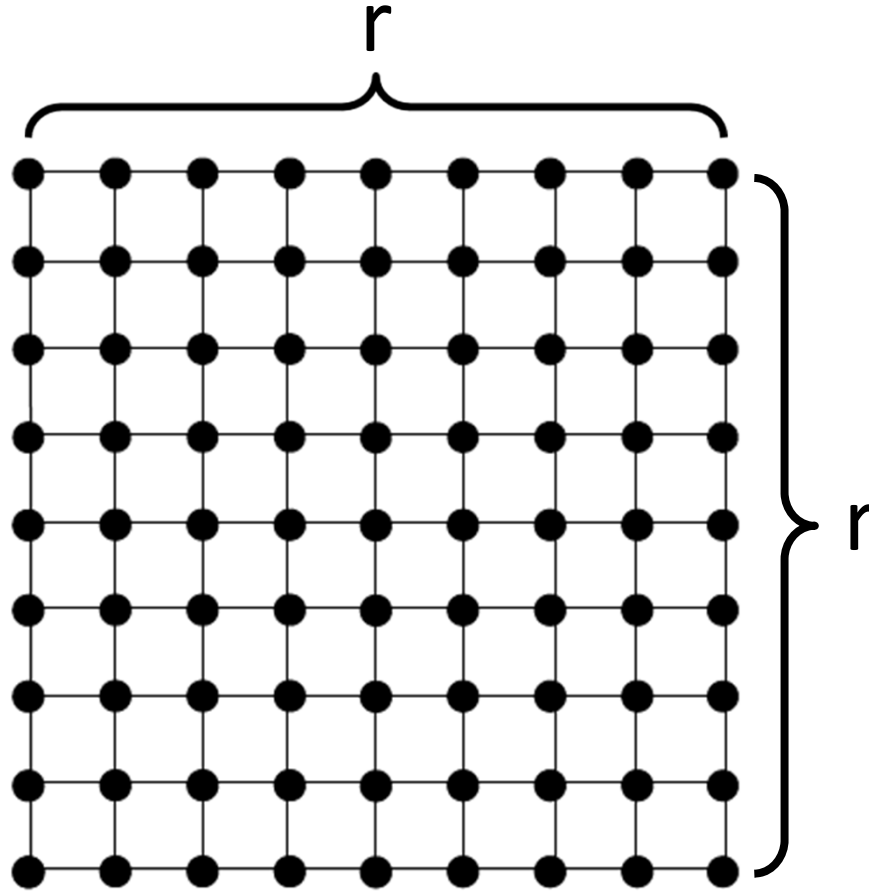
Conjecture: For every pair  $a, b$  of integers with  $2 \leq a \leq b$ , there exists a connected graph  $G$  such that  $\dim(G)=a$  and  $\dim^+(G)=b$ .

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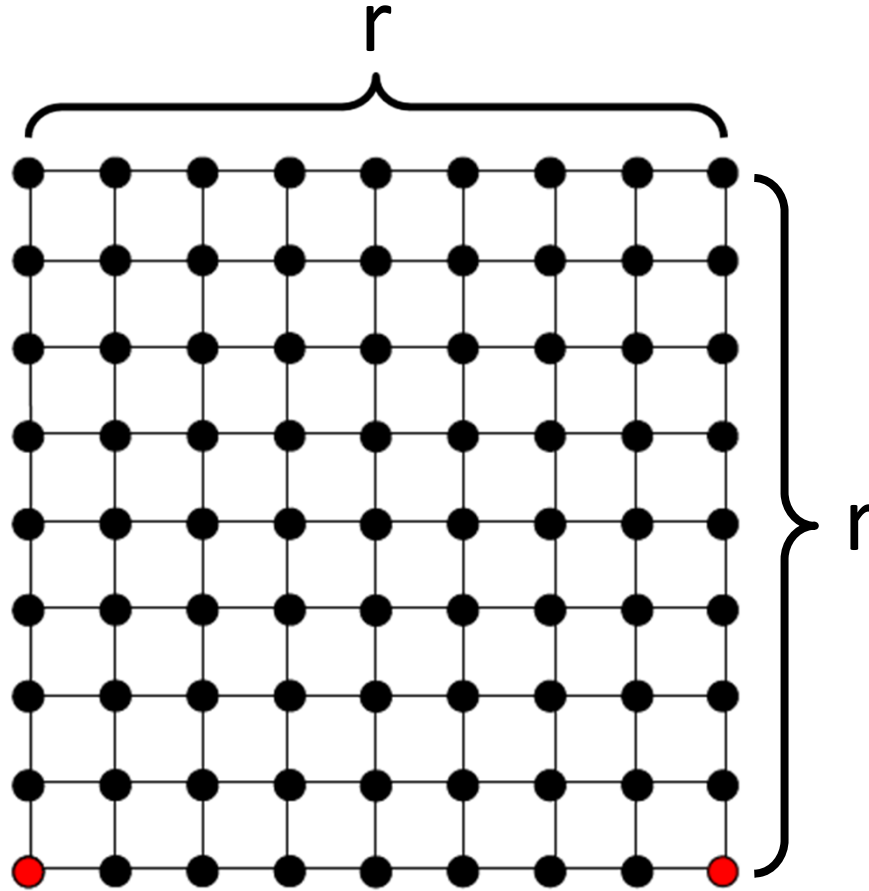
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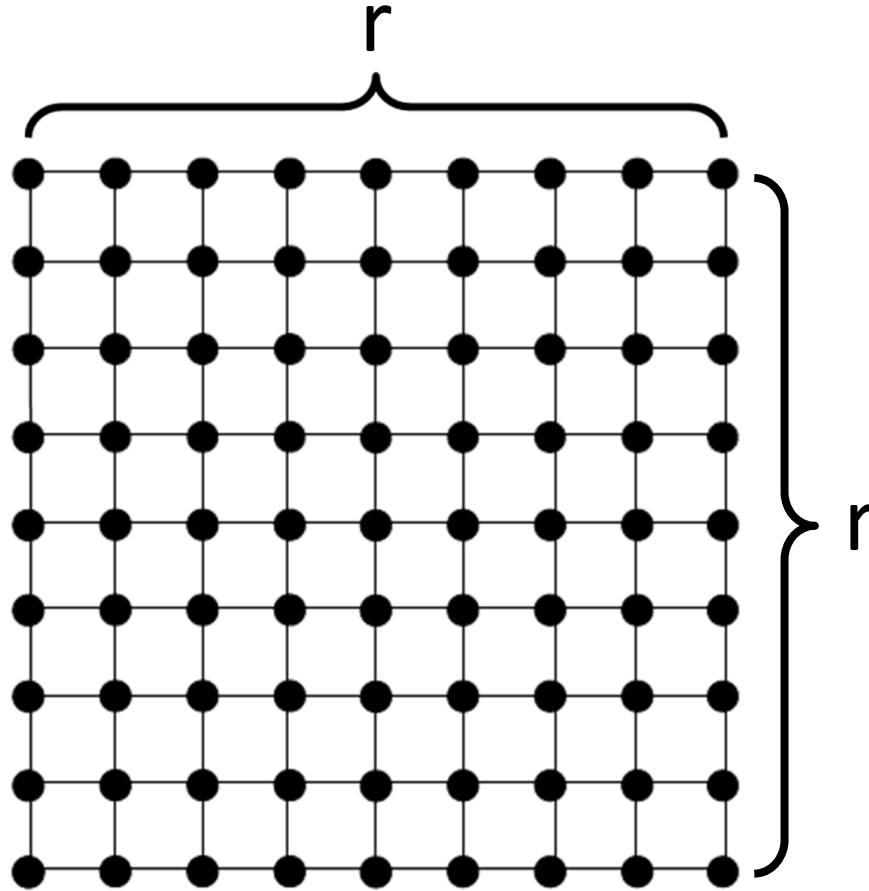
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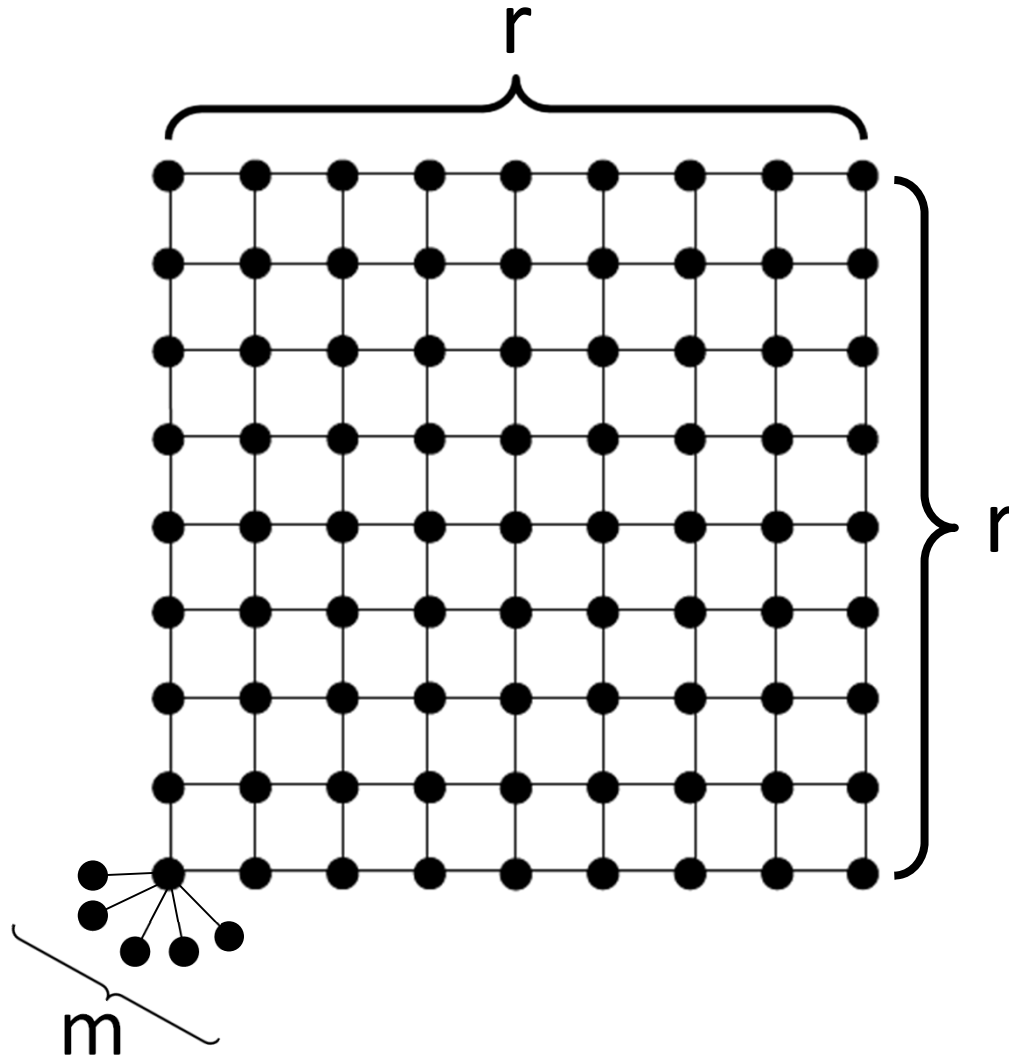
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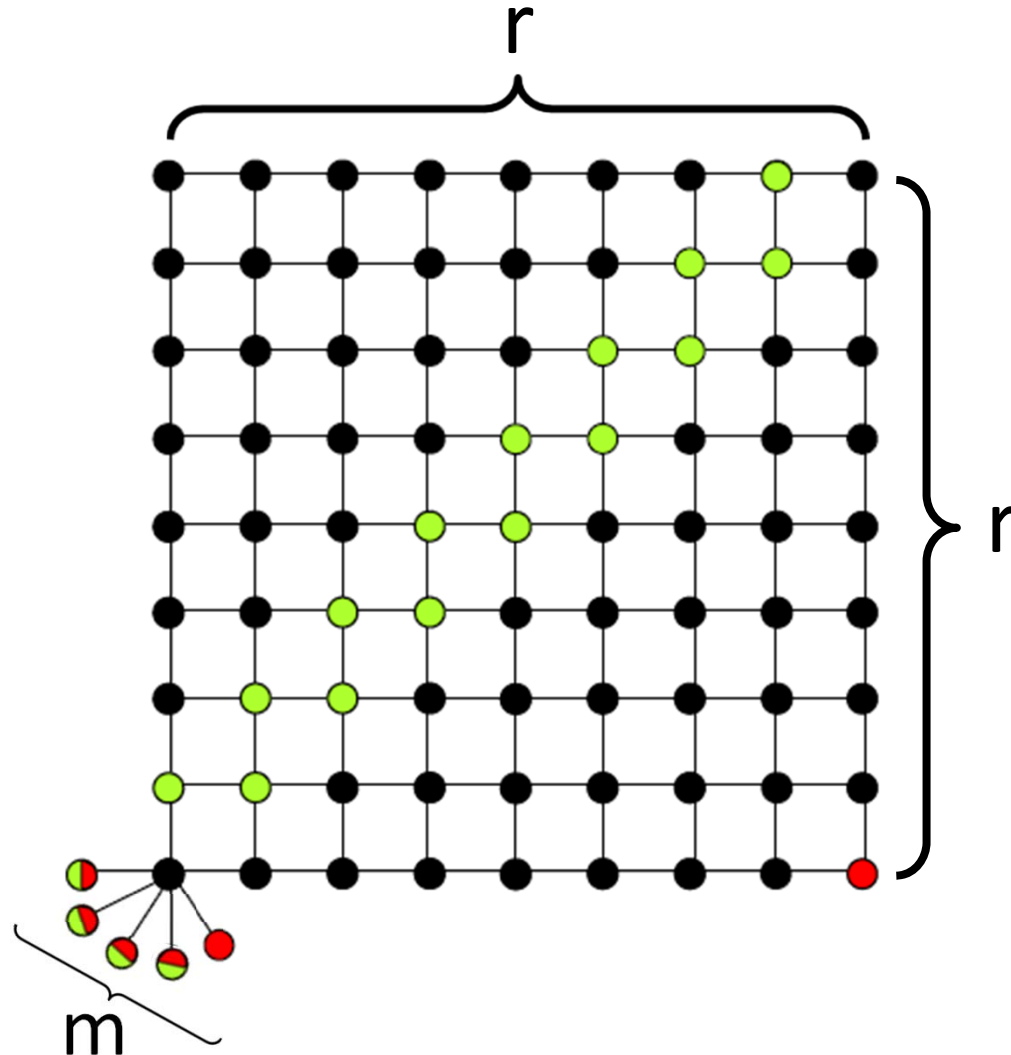
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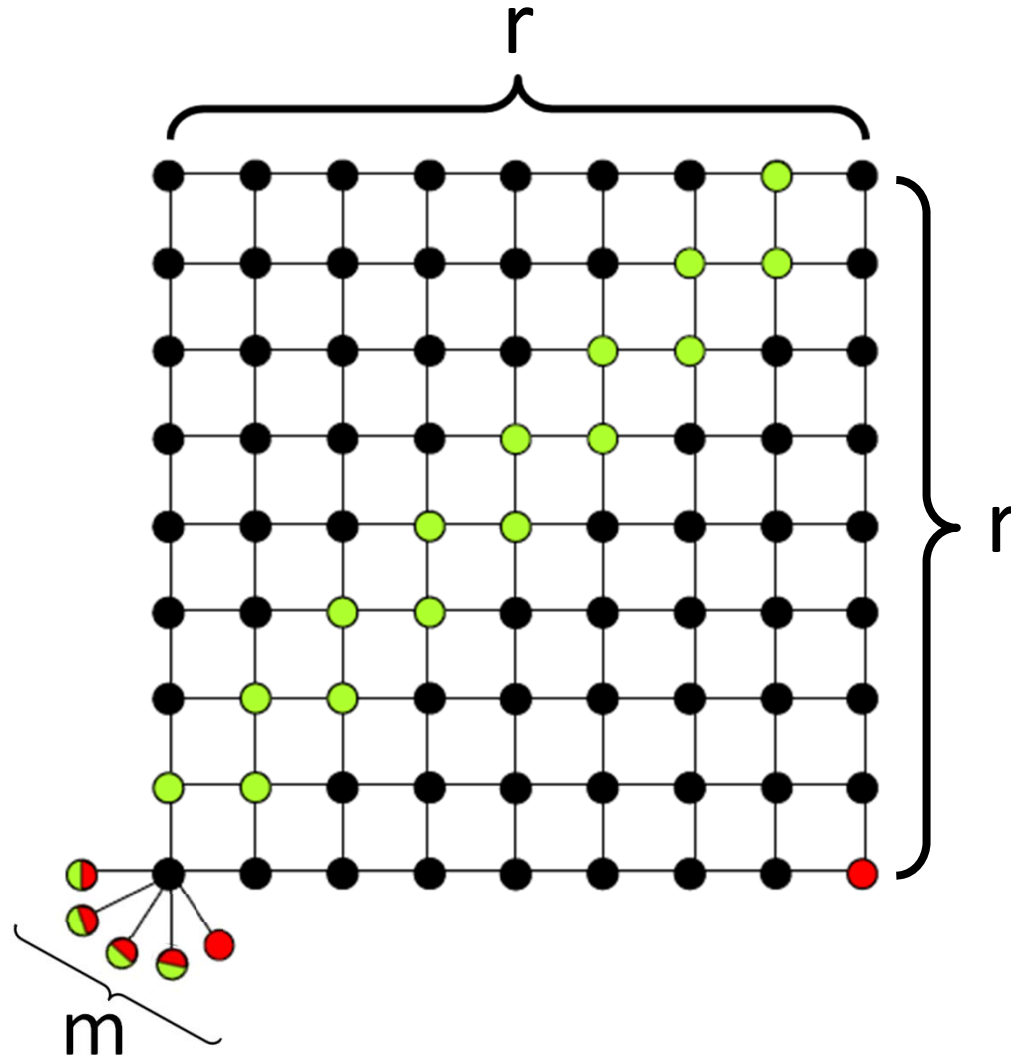
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Theorem: For every pair  $a, b$  of integers with  $2 \leq a \leq b$ , there exists a connected graph  $G$  such that  $\dim(G) = a$  and  $\dim^+(G) = b$ .



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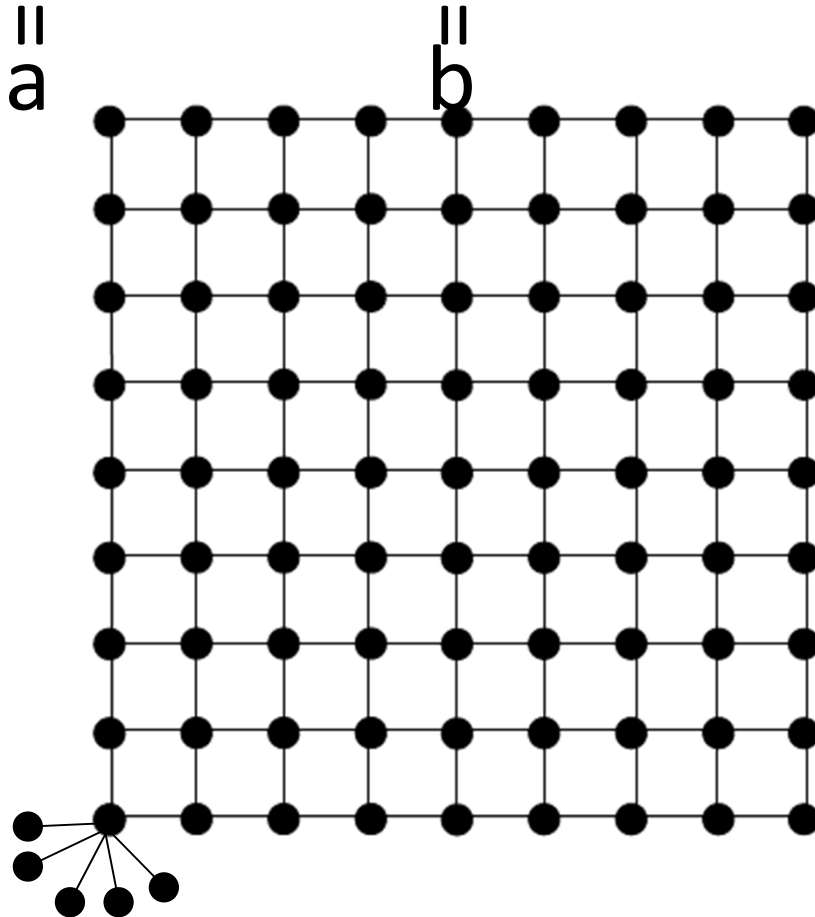
$\parallel$   
a

$\parallel$   
b



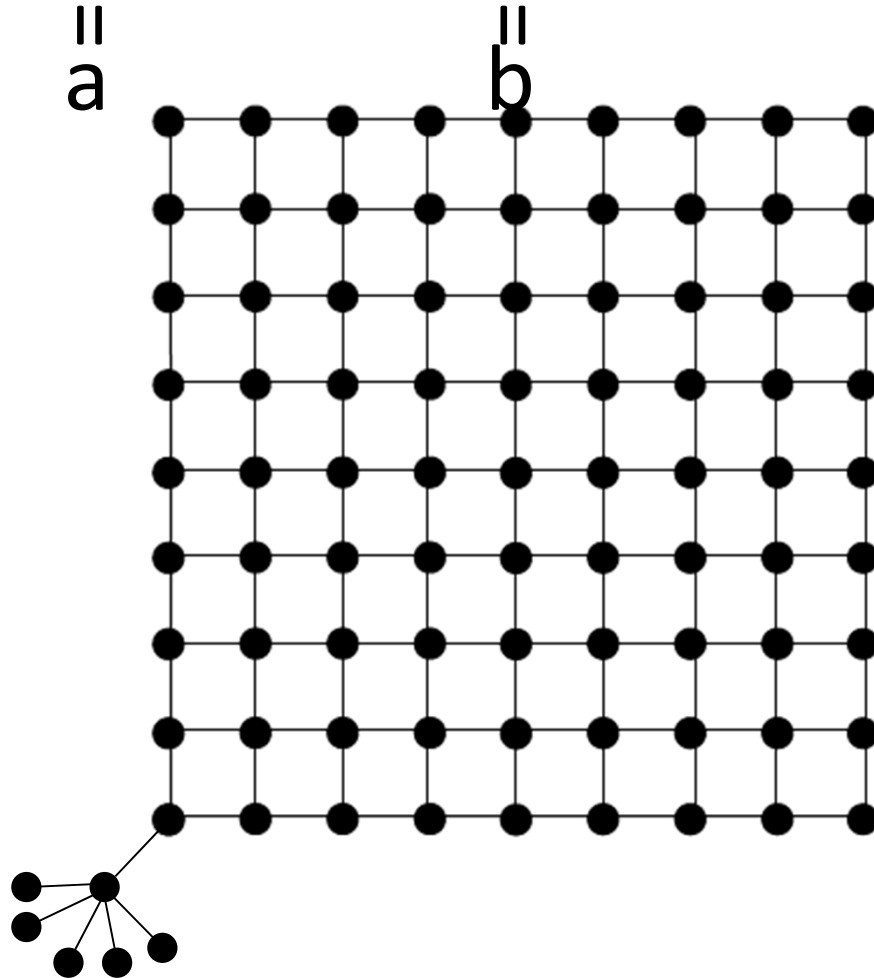
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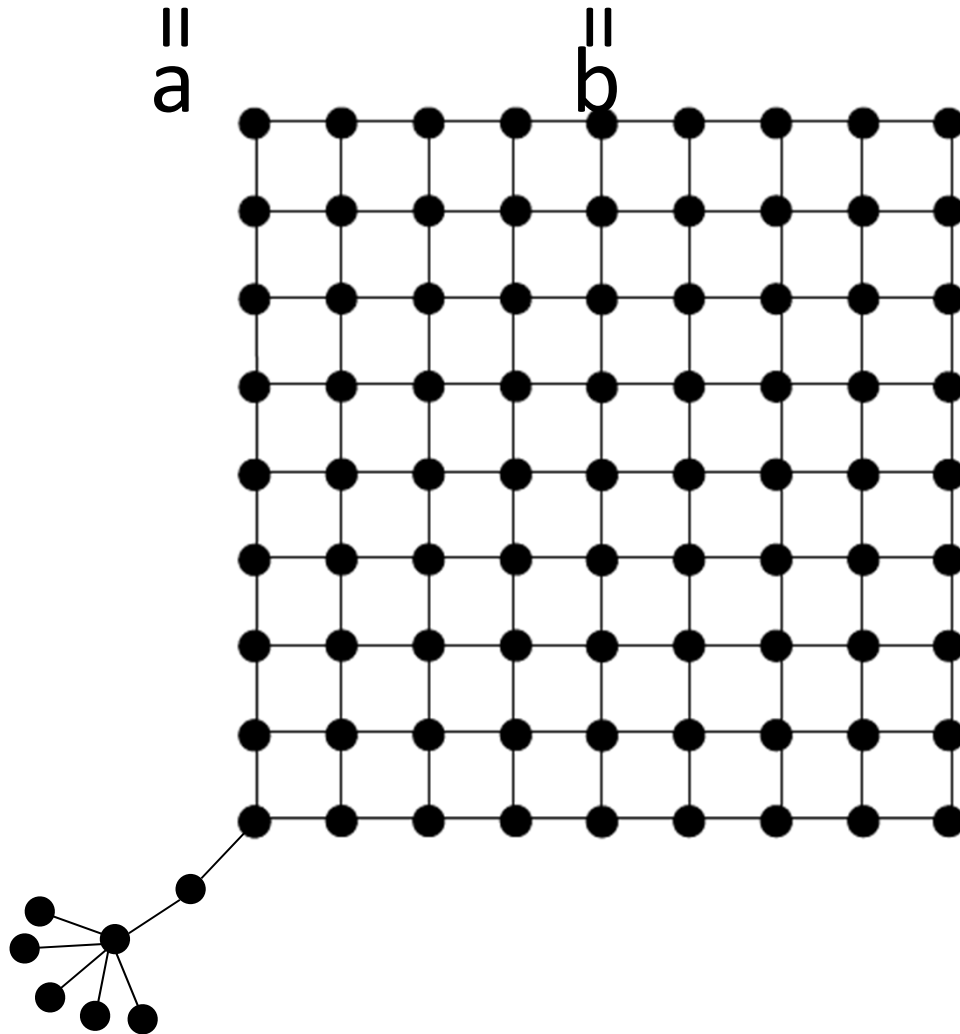
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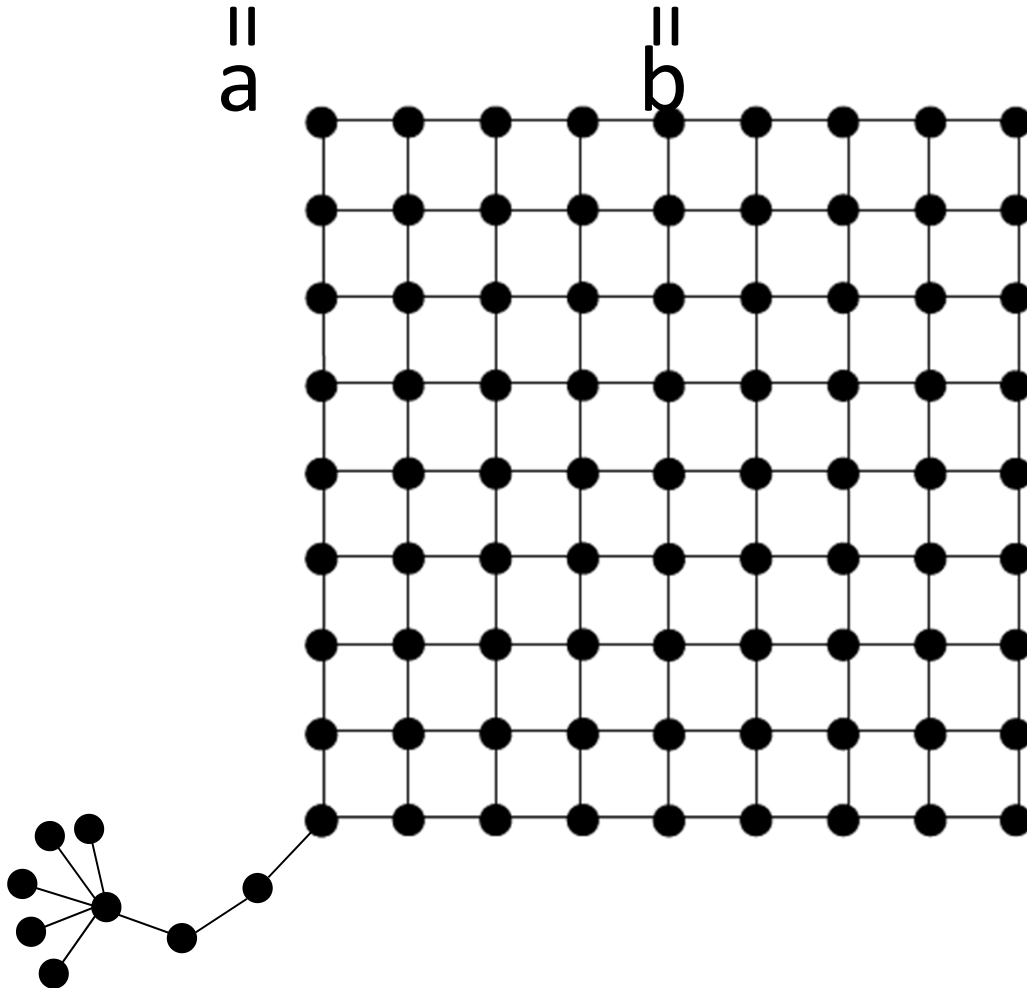
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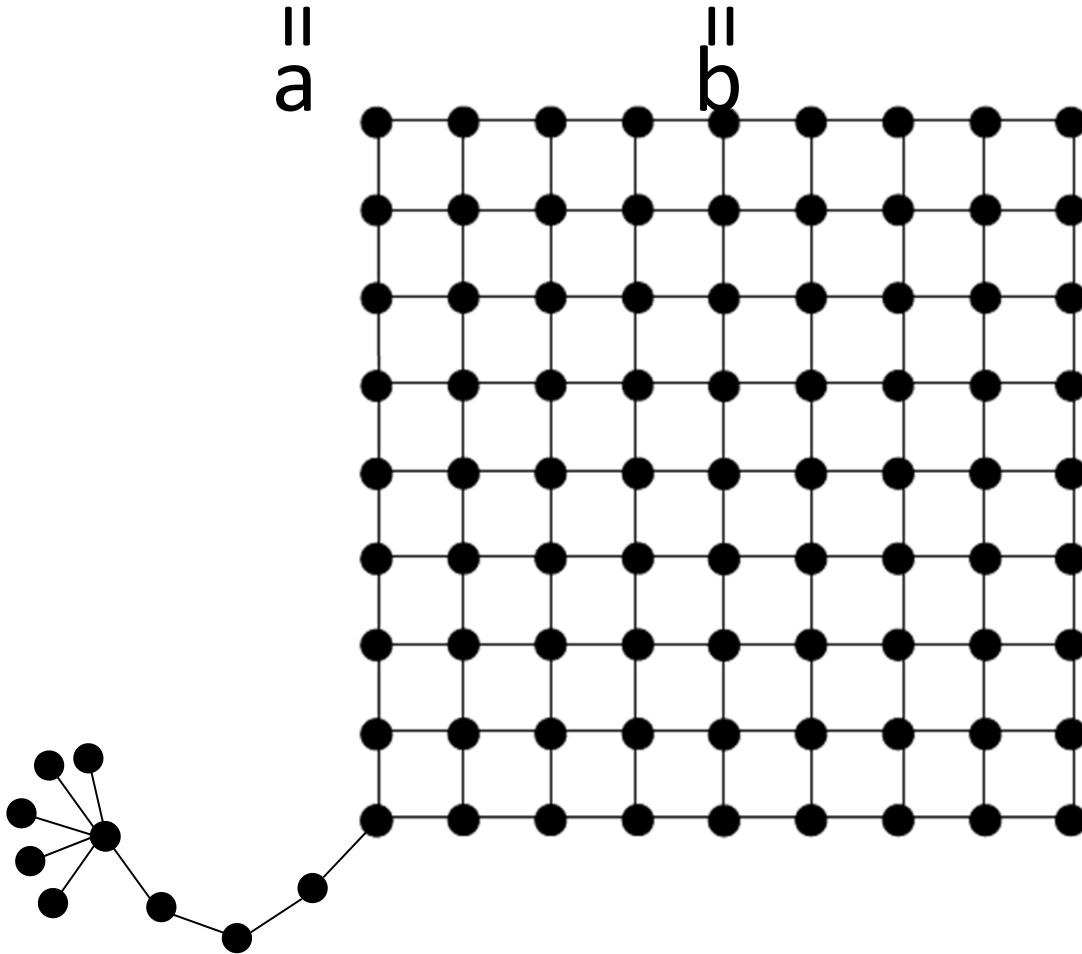
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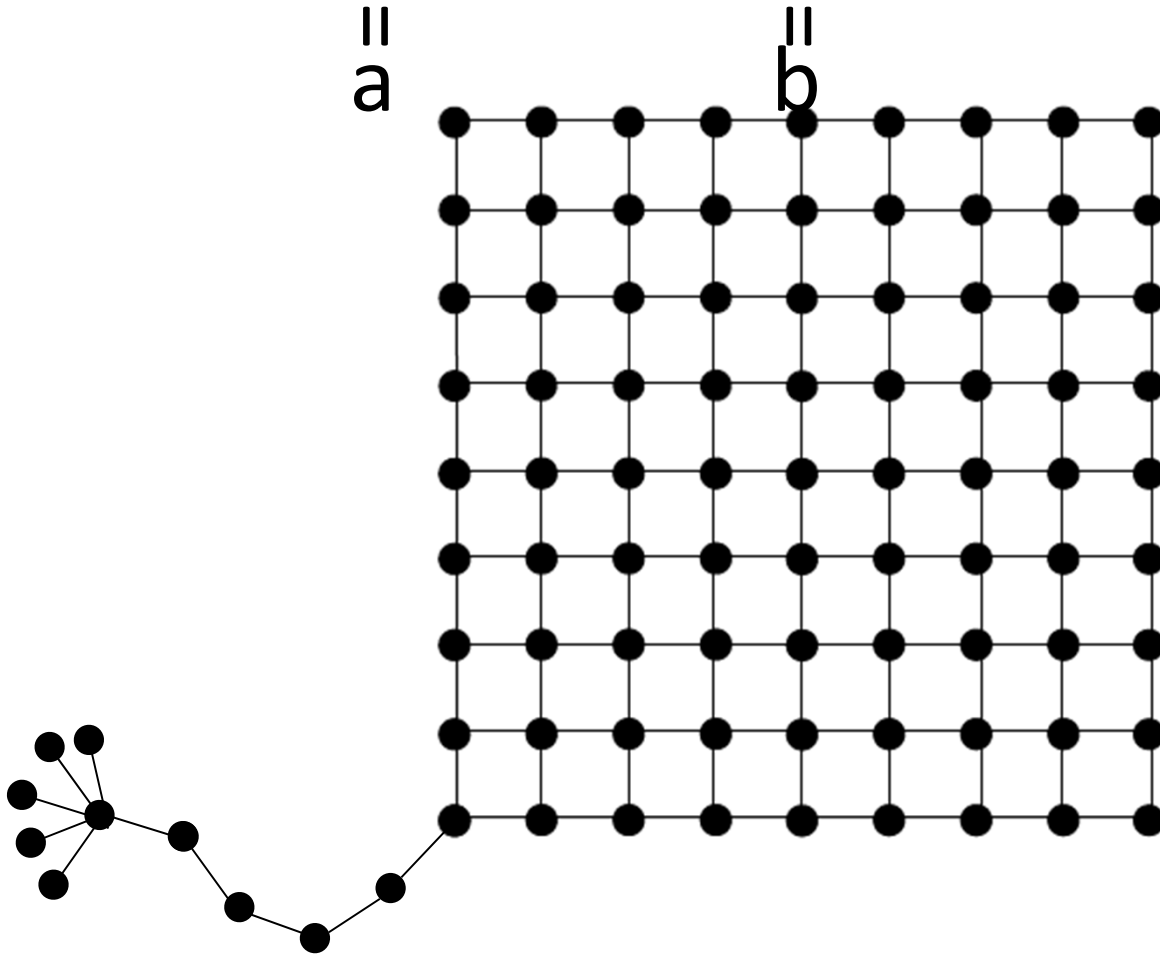
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$\parallel$   
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$\parallel$   
b

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$$\dim(G) \leq \dim^+(G) \leq \text{res}(G)$$

||  
C



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||  
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[Garijo, G., Márquez]  $D \leq 3\text{res} - 5$  (res > 3)

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$\parallel$   
 $\mathbb{C}$

$$[\text{KhRaRo}'96] \quad n \leq D^{\dim} + \dim$$

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QUESTION: Realization of triples  $(a,b,c)$ .

**THANKS!**