On k-domination and j-dependence in graphs

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Let G be a graph and let k and j be positive integers. A subset D of the vertex set of G is a k-dominating set if every vertex not in D has at least k neighbors in D. The k-domination number $\gamma_k(G)$ is the cardinality of a smallest k-dominating set of G. A subset $I \subseteq V(G)$ is a j-dependent set of G if every vertex in I has at most j-1 neighbors in I. The j-dependence number $\alpha_j(G)$ is the cardinality of a largest j-dependent set of G. In this work, we study the interaction between $\gamma_k(G)$ and $\alpha_j(G)$ in a graph G. Hereby, we generalize some known inequalities concerning these parameters and put into relation different known and new bounds on k-domination and j-dependence. Finally, we will discuss several consequences that follow from the given relations, while always highlighting the symmetry that exists between these two graph invariants.