



Bordeaux Workshop on Identifying Codes and related topics

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Booklet of abstracts

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Some useful definitions

Separating set (also known as *separating code*): subset C of $V(G)$ such that for each pair x, y of vertices of G , $B_1(x) \cap C \neq B_1(y) \cap C$.

Identifying code: subset C of $V(G)$ that is both a dominating set and a separating code of G .

Locating-dominating set (also known as *locating-dominating code*): subset C of $V(G)$ that is both a dominating set and for each pair x, y of vertices of $V(G) \setminus C$, $B_1(x) \cap C \neq B_1(y) \cap C$.

Variants on the previous concepts can be defined. For example, an $(r, \leq \ell)$ -*identifying code* is a distance- r dominating set C of G such that for each pair X, Y of subsets of vertices of G of cardinality at most ℓ , $B_r(X) \cap C \neq B_r(Y) \cap C$, where for $S \subseteq V(G)$, $B_r(S) = \cup_{s \in S} B_r(s)$. Similar concepts can also be defined in digraphs by replacing balls by incoming balls.

Metric basis (also known as *locating set*): subset C of $V(G)$ such that for each pair x, y of vertices of G , there is a vertex $c \in C$ such that $d(c, x) \neq d(c, y)$. The minimum cardinality of a metric basis is the *metric dimension*.

Locating colouring: proper vertex-colouring of G such that for each pair x, y of vertices of G , there is a colour C such that the minimum distance from x to a vertex coloured with C is different from the minimum distance from y to a vertex coloured with C .

Locally identifying colouring: proper vertex-colouring f of G such that for each pair x, y of adjacent vertices of G with $B_1(x) \neq B_1(y)$, $\cup_{v \in B_1(x)} f(v) \neq \cup_{w \in B_1(y)} f(w)$.

Discriminating code of a bipartite graph with parts A and B : subset C of A such that for each pair x, y of vertices in B , $B_1(x) \cap C \neq B_1(y) \cap C$.

Monday talks

Distinguishing sets in graphs

by Peter Slater

Abstract: A collection $S = \{S(1), S(2), \dots, S(t)\}$ of subsets of the vertex set $V(G)$ is distinguishing if every vertex is contained in at least one $S(i)$ and, for every two distinct vertices, some $S(i)$ contains exactly one of them. Distinguishing collections are of interest in finding an intruder or faulty node in a system modeled by a graph. Various situations in which a detection device at a vertex v can detect an intruder in its closed neighborhood $N[v]$ will be considered:

- (1) $\{v\}$ and $N(v)$ – locating-dominating sets,
- (2) $N[v]$ – identifying codes, and
- (3) $N(v)$ – open-locating-dominating sets.

Various other distinguishing collections of sets are also of interest.

With emphasis on open problems, a (partial) historical overview of these topics will be presented. Thinking of each $S(i)$ as a detector which can determine if the intruder is in set $S(i)$, various conditions under which detectors can be faulty will be discussed.

Extensional acyclic digraphs and set graphs

by Alexandru Tomescu with Martin Milanic and Romeo Rizzi

Abstract: A digraph is called ‘extensional’ if its vertices have pairwise distinct (open) out-neighborhoods. Extensional acyclic digraphs originate from set theory where they represent transitive closures of hereditarily finite sets. We will present some results concerning the underlying graphs of extensional acyclic digraphs, which we call ‘set graphs’. Even though we argue that recognizing set graphs is NP-complete, we show that set graphs contain all connected claw-free graphs and all graphs with a Hamiltonian path. In the case of claw-free graphs, we provide a polynomial-time algorithm for finding an extensional acyclic orientation.

Extensional digraphs can also be characterized in terms of a slight variation of the notion of separating code, which we call open-out-separating code. Concerning this, deciding if a digraph admits an open-out-separating code of given size is NP-complete.

Regular partial linear spaces admitting identifying codes

by Camino Balbuena with Gabriela Araujo-Pardo, Luis Pedro Montejano and Juan Carlos Valenzuela

Abstract: A characterization of k -regular partial linear spaces admitting a $(1, \leq k)$ -identifying code is given. As the incidence graph of a k -regular partial linear space is a k -regular bipartite graph of girth at least six, we derive a characterization of such a graphs admitting a $(1, \leq k)$ -identifying code. We describe a family of k -regular partial linear spaces on $2(k-1)^2 + k$ points and $2(k-1)^2 + k$ lines which do not admit a $(1, \leq k)$ -identifying code; and finally it is shown that the smallest $(k; 6)$ -graphs (known up to now), where $k-1$ is not a prime power, admit a $(1, \leq k)$ -identifying code.

Locally identifying colouring of graphs

by Aline Parreau with Louis Esperet, Florent Foucaud, Sylvain Gravier, Iiro Honkala, Tero Laihonen, Mickaël Montassier, Pascal Ochem and Guillem Perarnau

Abstract: We introduce the notion of locally identifying colouring (lid-colouring for short) : a proper colouring of vertices is said to be identifying if for any pair of adjacent vertices u and v , the set of colours in the closed neighbourhood of u and v are distinct (when it is possible).

The minimum number of colours needed for a graph G to have a lid-colouring is called the lid-chromatic number. We show that the lid-chromatic number of a graph G is at most $O(D^2)$ where D is the maximum degree of G . We also study the problem in some class of graphs, in particular in subclasses of chordal graphs and in planar graphs.

Efficient graph structures for vertex identification

by *Julien Moncel*

Abstract: Given a graph G admitting no twins, the problem of finding an identifying code of G is known to be NP-complete. In addition, if G has n vertices, we know that the minimum cardinality of such a code lies within the interval $[\lceil \log_2(n+1) \rceil, n-1]$. In some sense, the graphs for which only $\lceil \log_2(n+1) \rceil$ vertices among n are required to perform vertex identification are efficient. Indeed, if we had the choice of a graph structure to perform vertex identification, then we would surely choose such a graph.

In this talk, we focus on graph structures for which the minimum cardinality of an identifying code is small, compared to the total number of vertices.

We first recall the results of Charon, Hudry and Lobstein, that show that for every number in the interval $[\lceil \log_2(n+1) \rceil, n-1]$ there exists a graph requiring exactly this number of vertices for vertex identification. This result is true even for identification at distance $r > 1$.

In the second part of the talk we discuss about the case of the identification of sets of vertices. In this more general setting, there still remains open questions. In particular, the interval within which lies the cardinality of an identifying code is not precisely known.

We will furthermore discuss the link between these codes and the so-called superimposed codes, introduced by Kautz and Singleton in the sixties. We will show how, in special cases, one can use superimposed codes to construct identifying codes. We will also address the question of providing explicit constructions of such codes.

Tuesday talks

A generalization of identifying codes, the watching systems

by Antoine Lobstein with David Auger, Irène Charon and Olivier Hudry

Abstract: We introduce the notion of watching systems in graphs, which is a generalization of that of identifying codes. We give some basic properties of watching systems, an upper bound on the minimum size of a watching system, and results on the graphs which achieve this bound; we also study the cases of the paths and cycles, and give complexity results.

Watching systems in the king grid

by David Auger with Iiro Honkala

Abstract: We consider the infinite king grid where we investigate properties of watching systems, an extension of the notion of identifying code.

The locating and locating total bondage number for some graphs

by *Widad Dali with Mostafa Blidia*

Abstract: A locating-dominating set of a graph $G = (V, E)$ is a dominating set $S \subseteq V$ such that for every pair of distinct vertices u and v in $V - S$, $N(u) \cap S \neq N(v) \cap S$. The minimum cardinality of a locating-dominating set is denoted by $\gamma_L(G)$. A locating- total dominating set of a graph $G = (V, E)$ with $\delta(G) \geq 1$, is a locating- dominating set S such that the induced subgraph $\langle S \rangle$ is without isolated vertex. The minimum cardinality of a locating-total dominating set is denoted by $\gamma_L^t(G)$. The locating bondage number $b_L(G)$ (respect, the locating total bondage number $b_L^t(G)$) is the cardinality of a smallest set $E' \subseteq E$ (respect, $E' \subset E$) for which $\gamma_L(G - E') > \gamma_L(G)$ (respect, $\delta(G - E') \geq 1$ and $\gamma_L^t(G - E') > \gamma_L^t(G)$). In this paper we study the locating and locating total bondage number for some graphs.

On the identification of vertices in graphs using paths

by Matjaž Kovše with Florent Foucaud

Abstract: A set $\mathcal{C} = \{C_1, C_2, \dots, C_k\}$ of subgraphs of a graph G is said to identify the vertices of G if all sets $I(v, \mathcal{C}) = \{i \mid v \in C_i\}$ are nonempty and for any pair of distinct vertices u, v of G , $I(u, \mathcal{C}) \neq I(v, \mathcal{C})$. The idea here is that from knowing all sets $I(v, \mathcal{C})$ we can uniquely determine the vertices of G .

If we restrict \mathcal{C} to be a subset of a given family \mathcal{G} of subgraphs of G , we denote by $ID_{\mathcal{G}}(G)$, the minimum number of subgraphs of G from \mathcal{G} needed to identify all vertices of G . If we take as \mathcal{G} to be the family of all maximal stars of G , we get the definition of identifying codes. The case when \mathcal{G} equals the family of all cycles or closed walks of G , has been studied for special graph classes by Rosendahl and Honkala, Karpovsky, Litsyn.

In this talk, we present some results when \mathcal{G} equals the family \mathcal{P} of all paths of a graph G . In particular, we determine $ID_{\mathcal{P}}(T)$ for any tree T in terms of the number of leaves and the number of vertices of degree 2. For a path on n vertices, P_n , $ID_{\mathcal{P}}(P_n) = \lceil \frac{n+1}{2} \rceil$. For trees without vertices of degree 2 we have $ID_{\mathcal{P}}(T) = \lceil \frac{2l}{3} \rceil$. When a tree T has also vertices of degree 2 more cases appear and we present all of them.

Results about the extremal cases in terms of the number of vertices for the value of $ID_{\mathcal{P}}(G)$ for general graphs will also be discussed.

Structural properties of identifying codes: some results, conjectures and open problems

by Olivier Hudry with David Auger, Irène Charon, Iiro Honkala and Antoine Lobstein

Abstract: Let G be a simple, undirected graph with vertex set V . For any vertex v of V and any integer r greater than or equal to 1, we denote by $B(r, v)$ the ball of radius r and centre v . A subset C of V is said to be an r -identifying code in G if the intersections of $B(r, v)$ and C , for v in V , are all nonempty and pairwise distinct. A graph G admitting an r -identifying code is called r -twin-free, and in this case the size of a smallest r -identifying code in G is denoted by $c(G)$.

We study the following structural problem: let G be an r -twin-free graph, and G^* be a graph obtained from G by adding or deleting a vertex or an edge. If G^* is still r -twin-free, we compare the behaviours of $c(G)$ and $c(G^*)$, establishing results on their possible differences and ratios. We state also some conjectures or open problems dealing with these issues. More generally, we recall some conjectures or open problems related to identifying codes, based on joint works done with David Auger, Irène Charon, Iiro Honkala and Antoine Lobstein.

Wednesday talks

Codes for locating sets of vertices in a graph

by Tero Laihonen

Abstract: In this talk, we concentrate on identifying codes, which can locate more than one vertex in a graph. In particular, we will introduce known results in the case of binary hypercubes, infinite square grid, triangular grid, king grid and hexagonal mesh. We also study the extremal question of minimizing the number of vertices of a graph admitting an identifying code, which can simultaneously locate several vertices. We also discuss the case where we do not know in advance an upper bound on the number of vertices to be located.

Improved lower bound for 2-identifying code in the square grid

by Ville Junnila

Abstract: Previously, r -identifying codes have been studied in various grids. In particular, it has been shown that there exists a 2-identifying code in the square grid with density $5/29 \approx 0.172$ and that there are no 2-identifying codes with density smaller than $3/20 = 0.15$. Recently, the lower bound has been improved to $6/37 \approx 0.162$ by Martin and Stanton (2010). In this presentation, we further improve the lower bound by showing that there are no 2-identifying codes in the square grid with density smaller than $6/35 \approx 0.171$.

Minimum identifying codes in circular grids of given height

by Marwane Bouznif with Julien Moncel and Myriam Preissmann

Abstract: Given an integer h , we present an algorithm which finds a formula that allows to know, in a constant number of elementary operations, a minimum identifying code on circular grids of height h and length l , for all l . This algorithm is a particular case of a generic algorithm we developed for address optimization problems on fasciagraphs and rotagraphs. A fasciagraph is a graph constituted by a structure which repeats itself along a path, a rotagraph is a graph constituted by a structure which repeats itself along a cycle. Hence these graphs generalize grids and circular grids, respectively.

We implemented this algorithm for a height 3, and furthermore, we used our results to compute the minimum density of an identifying code on the infinite band of height 3, which is lower than the previous best known bound.

Locating dominating codes versus locating-dominating codes

by Ignacio Pelayo with Jose Cáceres, Carmen Hernando, Mercè Mora and Maria Luz Puertas

Abstract: A dominating set $S = \{u_1, \dots, u_k\}$ of a connected graph G is called locating dominating if every vertex v is uniquely determined by the k -vector $(d(v, u_1), \dots, d(v, u_k))$. If moreover, every vertex v not in S is also uniquely determined by the set of neighbors of v belonging to S , then it is said to be locating-dominating. In this talk, we present our more significant contributions on both minimum locating dominating and locating-dominating sets, mainly concerning extreme values, realization theorems and Cartesian products.

Identifying codes in graphs of given maximum degree

by Florent Foucaud with Sylvain Gravier, Ralf Klasing, Adrian Kosowski, Reza Naserasr, Aline Parreau, Guillem Perarnau, André Raspaud and Petru Valicov

Abstract: We investigate upper bounds on the identifying code number for graphs with given maximum degree. We present a conjecture on this topic as well as various bounds for different graph classes.

Identifying codes in line graphs

by Petru Valicov with Florent Foucaud, Sylvain Gravier, Reza Naserasr and Aline Parreau

Abstract: We study edge-identifying codes, a concept which is equivalent to identifying codes in line graphs. We prove the NP-completeness of the edge-identifying code problem. Additionally, we give lower and upper bounds for the size of the minimum edge-identifying code.

Thursday talks

Domination and independent domination in direct products of graphs

by Sandi Klavžar

Abstract: The direct product of graphs is one of the four standard graph products. In many respects this product is the most difficult one, for instance (i) no general formula for its (edge-)connectivity as a function of factor graphs is known, and (ii) determining its chromatic number reduces to (dis)proving the outstanding Hedetniemi's conjecture.

In this talk we will survey what is known about domination and independent domination in direct products. A special emphasis will be given to idomatic partitions (that is, partitions into independent sets that are at the same time dominating sets) of direct products of complete graphs.

On k -domination and j -dependence in graphs

by *Adriana Hansberg with Ryan Pepper*

Abstract: Let G be a graph and let k and j be positive integers. A subset D of the vertex set of G is a k -dominating set if every vertex not in D has at least k neighbors in D . The k -domination number $\gamma_k(G)$ is the cardinality of a smallest k -dominating set of G . A subset $I \subseteq V(G)$ is a j -dependent set of G if every vertex in I has at most $j-1$ neighbors in I . The j -dependence number $\alpha_j(G)$ is the cardinality of a largest j -dependent set of G . In this work, we study the interaction between $\gamma_k(G)$ and $\alpha_j(G)$ in a graph G . Hereby, we generalize some known inequalities concerning these parameters and put into relation different known and new bounds on k -domination and j -dependence. Finally, we will discuss several consequences that follow from the given relations, while always highlighting the symmetry that exists between these two graph invariants.

Sufficient conditions for the existence of two disjoint independent dominating sets

by *Oliver Schaudt*

Abstract: We present some sufficient conditions for the existence of two disjoint independent dominating sets. First, we give a sufficient condition in terms of forbidden induced subgraphs, using a connection to the theory of structural total domination. Second, we prove that every 2-terminal series-parallel graph has two disjoint independent sets.

Domination game on graphs

by Douglas Rall

Abstract: In this talk we present a domination game on finite graphs that was recently introduced by Brešar, Klavžar and Rall, *Domination game and an imagination strategy*, SIAM J. Discrete Math., Vol 24, No. 3, 979–991 (2010). Two players, Dominator and Staller, alternate choosing a vertex from a graph G . Each vertex chosen by a player must dominate at least one new vertex that was not dominated by the set of vertices chosen prior to that in the game. Both players use an optimal strategy—Dominator to end the game as quickly as possible and Staller to prolong the game. Dominator starts Game 1, and the resulting number of vertices chosen when Game 1 ends is the game domination number of G , denoted $\gamma_g(G)$. When Staller has the first move on G (Game 2) the corresponding invariant is the Staller-start game domination number, $\gamma'_g(G)$.

We present most of what is known about this game and these two invariants including some known and conjectured bounds. In particular, even though the domination game is closely related to ordinary domination in some respects, we present some rather surprising results about the domination game on trees and spanning subgraphs. We also discuss the principal proof technique, the imagination strategy, and give a number of conjectures and open problems.

Friday talks

Locating and dominating sets in graphs

by Michael Henning

Abstract: In this talk, we present results on locating and dominating-type sets in graphs, including dominating sets, paired-dominating sets and total dominating sets. We consider such dominating sets which are also locating sets, that is distinct vertices in the graph are dominated by distinct subsets of the dominating set. We consider three variations of sets which are dominating (or paired-dominating or total dominating) and locating sets and investigate their properties.

Metric dimension on random geometric graphs

by Guillem Perarnau with Josep Díaz, Dieter Mitsche and Maria Serna

Abstract: Given a set of points in the unit square $[0, 1]^2$, a geometric graph is constructed by joining two points whether they lie at distance at most r . If the positions of these points follow a probability distribution we talk about random geometric graphs. They turned out to have many applications, such as modelling wireless sensor networks. We study the metric dimension of random geometric graphs.

Counting resolving sets in graphs

by Antonio González with Delia Garijo and Alberto Márquez

Abstract: This talk deals with three parameters related to resolvability in graphs: the metric dimension, the upper dimension and the resolving number. We first answer a question raised by Chartrand and Zhang asking for a characterization of the graphs with equal metric dimension and resolving number. We also solve in the affirmative a conjecture posed by Chartrand, Poisson and Zhang about the realization of the metric dimension and the upper dimension. Finally, we establish a relationship between the resolving number and the diameter of a graph, giving rise to a surprising result on the realization of the resolving number.

Identifying codes using Euclidean balls

by Aline Parreau with Ville Junnila and Tero Laihonen

Abstract: In this talk, we consider as sensors the set \mathbb{Z}^2 where one sensor can check its neighbours within Euclidean distance r . We construct tolerant identifying codes in this network that are robust against some changes in the neighbourhood monitored by each sensor.

Open neighborhood locating-dominating sets

by Peter Slater

Abstract: Vertex set $S = v_1, v_2, \dots, v_t$ in graph G is an open neighborhood locating-dominating set if $\{N(v_1), \dots, N(v_t)\}$ is distinguishing, that is, for each vertex v in $V(G)$ the open neighborhood $N(v)$ has a unique non-empty intersection with S . Introductory results and open problems will be presented.

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